

**Estudio Analítico - Gráfico
de los
Poliedros Arquimedianos**

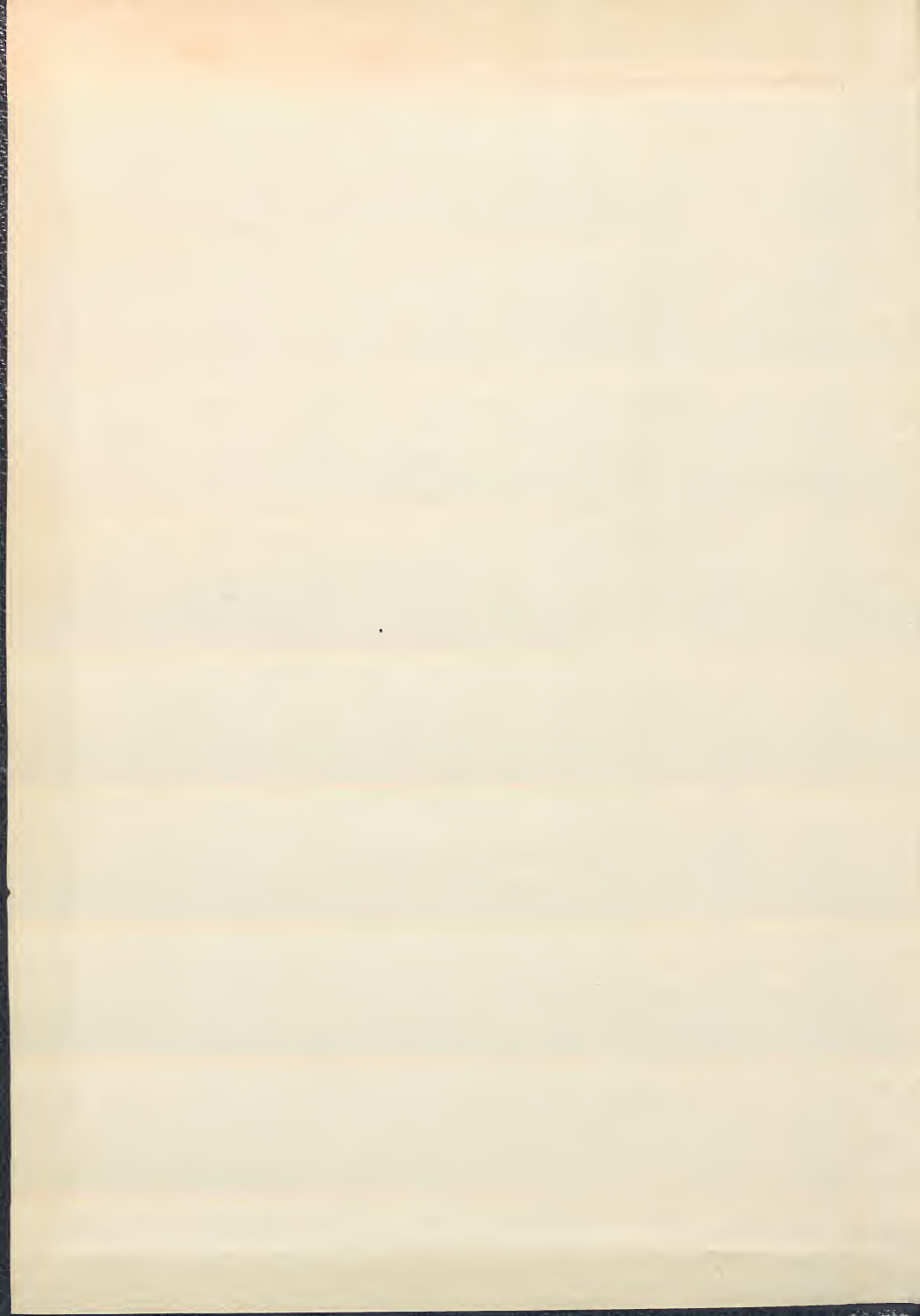
LAMINAS 44 AL 47

VIII

VIII

TA
514
LV-VI
00107505

Prof. T. Alvarez Peralto







R. 7815
 L1903099 x

ENUNCIADO

Representar por el método gráfico-analítico, en los planos I, II y III, el arquimedeano XII, en el que en cada vértice concurren un cuadrado, un hexágono y un decágono, todos regulares.

La longitud de su lado es de 14.5 mm, y las coordenadas de su centro O, son: O (72, 72, 85) mm.

Dibujar en formato A3V y a escala 1:1.

DATOS: O (72, 72, 85) mm

$l_{XII} = 14.5 \text{ mm}$

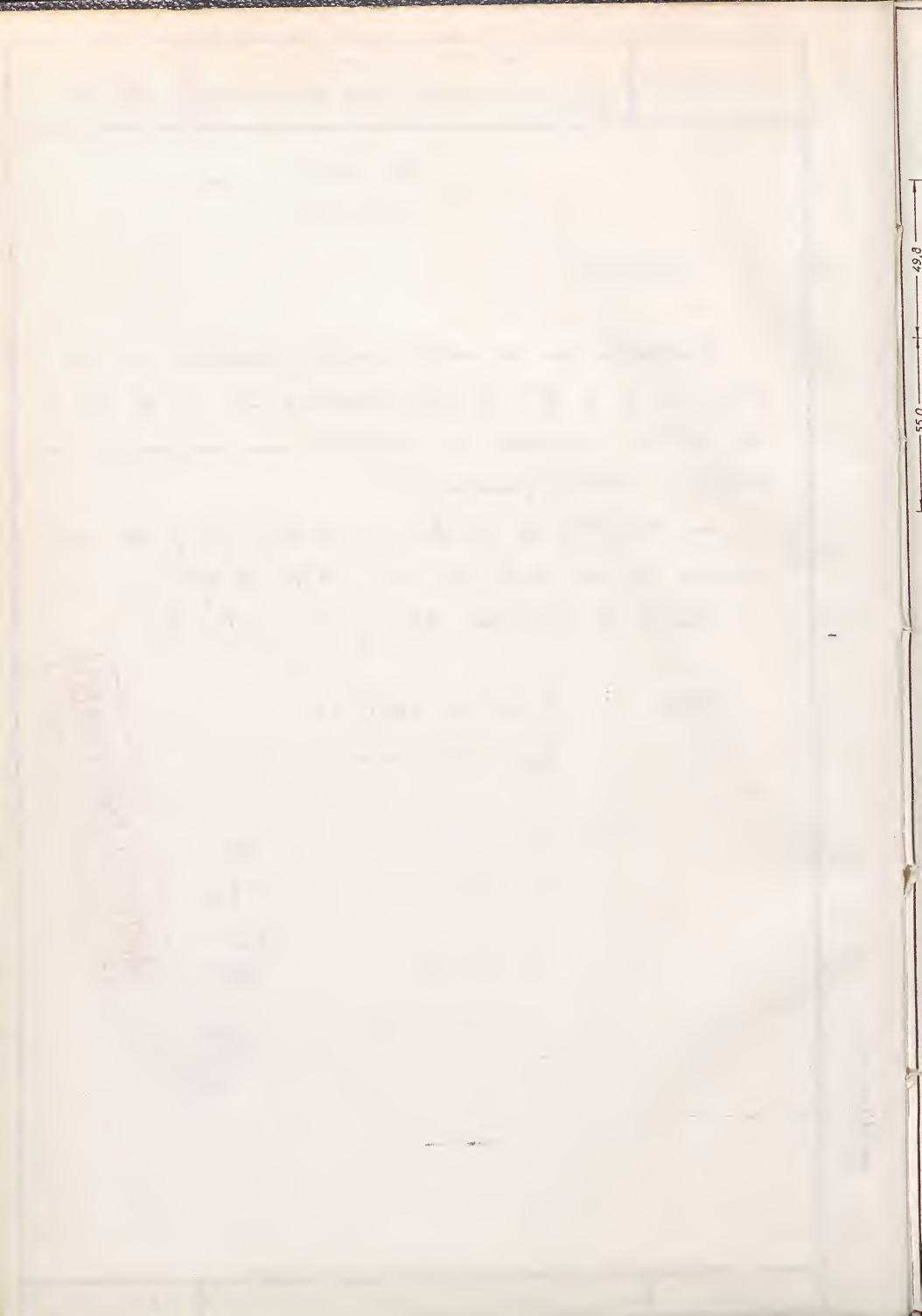
A

514

ALV

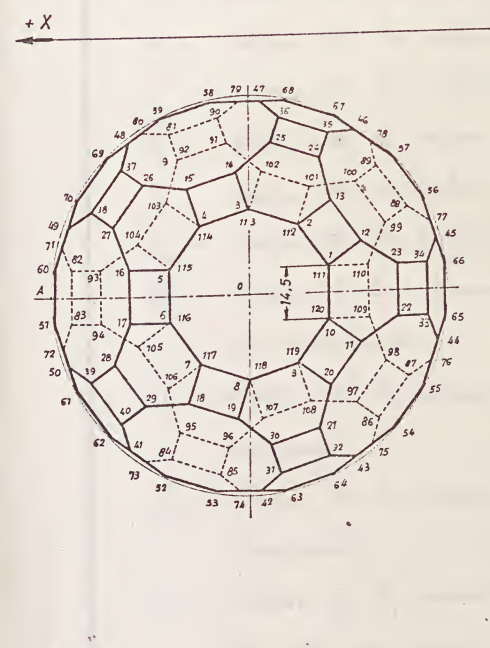
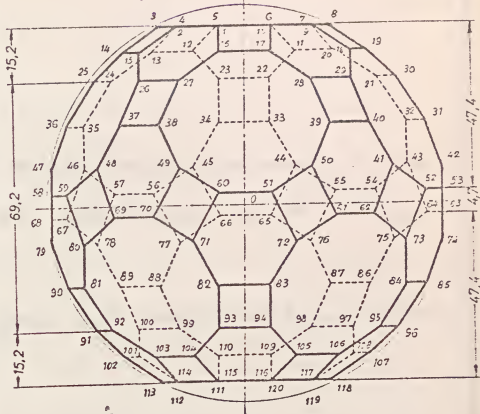
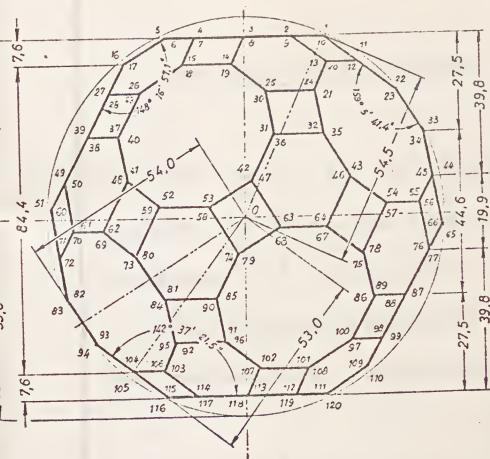
VIII







III



ARQUIMEDIANO XII

Número de caras cuadradas..... $C_4 = 30$
 Número de caras exagonales..... $C_6 = 20$
 Número de caras decagonales..... $C_{10} = 12$
 Número de vértices..... $V = 120$
 Número de aristas..... $A = 180$
 Número de caras de un ángulo sólido .. $1P_4 + 1P_6 + 1P_{10}$

ENUNCIADO

Representar por el método gráfico-analítico, en los planos I, II y III, el Arquimediano XII, en el que en cada vértice concurren un cuadrado, un exágono y un decágono, todos regulares.

La longitud de su lado es de su lado es de 14,5 mm. y las coordenadas de su centro O, son: O (72, 72, 85) mm.

Dibujar en formato A3v y a escala 1:1.

Propuesta	De entrega	Entregada	Calificación	(firma)	Escuela
Fecha:					Curso
Alumno:					
Escala	Arquimediano XII				Lámina 44
1:1					Curso 19 - 19



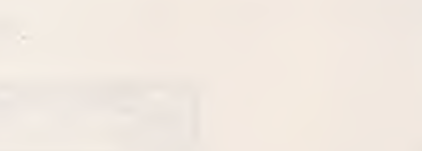
THE
SUN
AND
MOON
AND
STARS
AND
PLANETS
AND
COMETS
AND
METEORS
AND
AURORAE
AND
ECLIPSES
AND
OTHER
PHENOMENA
OF
HEAVEN



THE
SUN
AND
MOON
AND
STARS
AND
PLANETS
AND
COMETS
AND
METEORS
AND
AURORAE
AND
ECLIPSES
AND
OTHER
PHENOMENA
OF
HEAVEN



THE
SUN
AND
MOON
AND
STARS
AND
PLANETS
AND
COMETS
AND
METEORS
AND
AURORAE
AND
ECLIPSES
AND
OTHER
PHENOMENA
OF
HEAVEN



CONSIDERACIONES PREVIAS

Seguiremos en el estudio de este arquimedianos, las directrices y fórmulas generales planteadas en el "Arquimedianos I", lám. 33.

En el caso particular que nos ocupa, determinaremos las magnitudes siguientes:

- l = Arista del Arquimedianos XII (dato del ejercicio)
- a = Radio de la esfera circunscrita
- b = Radio de la esfera tangente a las aristas
- c_4 = Radio de la esfera tangente a las caras cuadradas.
- c_6 = Radio de la esfera tangente a las caras hexagonales
- c_{10} = Radio de la esfera tangente a las caras decagonales
- d_4 = Radio de la circunferencia circunscrita a una cara cuadrada.
- d_6 = Radio de la circunferencia circunscrita a una cara hexagonal
- d_{10} = Radio de la circunferencia circunscrita a una cara decagonal.
- m = Radio de la circunferencia circunscrita al polígono obtenido al unir los extremos de las aristas de un ángulo sólido.
- α = Ángulo rectilíneo del diedro formado por una

cara cuadrada, con el plano diametral del arquimedianos que pasa por una arista de aquella.

α_6 = Ángulo rectilíneo del diedro formado por una cara exagonal, con el plano diametral del arquimedianos que pasa por una arista de aquella.

α_{10} = Ángulo rectilíneo del diedro formado por una cara decagonal, con el plano diametral del arquimedianos que pasa por una arista de aquella.

φ_{4-6} = Ángulo rectilíneo del diedro formado por una cara cuadrada y otra exagonal.

φ_{4-10} = Ángulo rectilíneo del diedro formado por una cara cuadrada y otra decagonal.

φ_{6-10} = Ángulo rectilíneo del diedro formado por una cara exagonal y otra decagonal.

S = Superficie

V = Volumen

PROCESO GRÁFICO-ANALÍTICO

El estudio realizado de este arquimedianos, nos indica que se compone de 30 caras cuadradas, 20 caras exagonales y 12 caras decagonales; 120 vértices y 180 aristas.

En cada vértice concurren un cuadrado, un exágono y un decágono, todos regulares de igual lado "l".

Así pues, tendremos que:

ARQUIMEDIANO XII ($1P_4 + 1P_6 + 1P_{10}$); $C_4 = 30$; $C_6 = 20$; $C_{10} = 12$; $V = 120$; $A = 180$

Cálculo de sus magnitudes

Arista "l" del arquimedeano

Dato del ejercicio

Radio "m" de la circunferencia circunscrita al polígono obtenido al unir los extremos de las tres aristas que concurren en un ángulo sólido

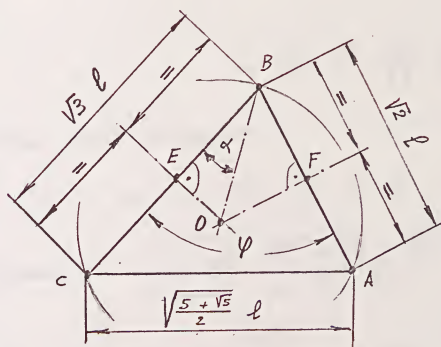


Figura 1

Este polígono es un triángulo A-B-C (fig. 1), escaleno, cuyos lados \overline{AB} , \overline{BC} y \overline{CA} , son respectivamente las diagonales que unen tres vértices consecutivos de un cuadrado, un hexágono y un decágono, todos re-

gulares y de lado "l".

Se demuestra en Geometría que estas diagonales son:

Subject: Mathematics

Chapter: Geometry

Topic: Area of a Triangle

Exercise: 1.1

Q.1. Find the area of a triangle whose base is 10 cm and height is 6 cm.

Sol: Given, base = 10 cm
 height = 6 cm
 Area of triangle = $\frac{1}{2} \times \text{base} \times \text{height}$
 $= \frac{1}{2} \times 10 \times 6$
 $= 5 \times 6$
 $= 30 \text{ cm}^2$



Q.2. Find the area of a triangle whose base is 12 cm and height is 8 cm.

a) En el cuadrado (diagonal) $\overline{AB} = \sqrt{2} \ell$ [1]

b) En el octágono (lado del triángulo inscrito) $\overline{BC} = \sqrt{3} \ell$ [2]

c) En el decágono (lado del pentágono regular inscrito) $\overline{CA} = \sqrt{\frac{5+\sqrt{5}}{2}} \ell$ [3]

De la figura 1, se deduce:

$$\overline{BO} = m = \frac{\overline{BE}}{\cos \alpha} = \frac{\overline{BF}}{\cos (\varphi - \alpha)} \quad [4]$$

de donde $\overline{BE} \times \cos (\varphi - \alpha) = \overline{BF} \times \cos \alpha$ "

$$\frac{\sqrt{3}}{2} \ell \cos (\varphi - \alpha) = \frac{\sqrt{2}}{2} \ell \cos \alpha \quad \sqrt{3} \cos (\varphi - \alpha) = \sqrt{2} \cos \alpha$$

$$\cos (\varphi - \alpha) = \frac{\sqrt{2}}{3} \cos \alpha \quad [5]$$

por otra parte tenemos:

$$\overline{AC}^2 = \overline{BC}^2 + \overline{BA}^2 - 2 \times \overline{BC} \times \overline{BA} \times \cos \varphi \quad \overline{BC}^2 + \overline{BA}^2 - \overline{AC}^2 = 2 \times \overline{BC} \times \overline{BA} \times \cos \varphi$$

de donde $\cos \varphi = \frac{\overline{BC}^2 + \overline{BA}^2 - \overline{AC}^2}{2 \times \overline{BC} \times \overline{BA}} = \frac{(\sqrt{3} \ell)^2 + (\sqrt{2} \ell)^2 - \left(\sqrt{\frac{5+\sqrt{5}}{2}} \ell\right)^2}{2 \times \sqrt{3} \ell \times \sqrt{2} \ell} =$

$$= \frac{3 + 2 - \frac{5+\sqrt{5}}{2}}{2 \sqrt{6}} = \frac{5 - \frac{5+\sqrt{5}}{2}}{2 \sqrt{6}} = \frac{5 - \sqrt{5}}{4 \sqrt{6}} = \frac{5 \sqrt{6} - \sqrt{30}}{4 \times 6} = \boxed{\frac{5 \sqrt{6} - \sqrt{30}}{24}} \quad [6]$$

pe [5] se deduce:

$$\sqrt{\frac{2}{3}} \cos \alpha = \cos (\varphi - \alpha) = \cos \varphi \cos \alpha + \sin \varphi \sin \alpha \quad [7]$$

1. Introduction

Date: _____
 Page: _____

The purpose of this study is to investigate the effect of the independent variable on the dependent variable. The study is designed to explore the relationship between the two variables and to determine the extent to which the independent variable influences the dependent variable. The study is conducted in a controlled environment to ensure the validity of the results.

The study is conducted in a controlled environment to ensure the validity of the results. The study is designed to explore the relationship between the two variables and to determine the extent to which the independent variable influences the dependent variable. The study is conducted in a controlled environment to ensure the validity of the results.

The study is conducted in a controlled environment to ensure the validity of the results. The study is designed to explore the relationship between the two variables and to determine the extent to which the independent variable influences the dependent variable. The study is conducted in a controlled environment to ensure the validity of the results.

Despejando de [7] " $\cos \varphi$ ", y sustituyéndolo en [4], tendremos (ver desarrollo de este cálculo en lám. 43, págs 4, 5)

$$m = \sqrt{\frac{5 - 6\sqrt{\frac{2}{3}} \cos \varphi}{4(1 - \cos^2 \varphi)}} \times l \quad [8]$$

en la que " $\cos \varphi$ " está dado por la expresión [6]. Para calcular m , tendremos:

a) Para el numerador del radical:

$$\begin{aligned} 5 - 6\sqrt{\frac{2}{3}} \cos \varphi &= 5 - 6\sqrt{\frac{2}{3}} \times \frac{5\sqrt{6} - \sqrt{30}}{24} = 5 - 6 \times \frac{\sqrt{2}(5\sqrt{6} - \sqrt{30})}{\sqrt{3} \times 24} = \\ &= 5 - 6 \times \frac{5\sqrt{12} - \sqrt{60}}{24\sqrt{3}} = 5 - \frac{5 \times 2\sqrt{3} - 2\sqrt{15}}{4\sqrt{3}} = 5 - \frac{5\sqrt{3} - \sqrt{15}}{2\sqrt{3}} = 5 - \frac{15 - \sqrt{45}}{6} = \\ &= 5 - \frac{15 - 3\sqrt{5}}{6} = \frac{30 - 15 + 3\sqrt{5}}{6} = \frac{15 + 3\sqrt{5}}{6} = \boxed{\frac{5 + \sqrt{5}}{2}} \end{aligned}$$

b) Para el denominador del radical:

$$\begin{aligned} 4(1 - \cos^2 \varphi) &= 4 \times \left(1 - \left(\frac{5\sqrt{6} - \sqrt{30}}{24}\right)^2\right) = 4 \times \left(1 - \frac{25 \times 6 + 30 - 10\sqrt{180}}{24^2}\right) = \\ &= 4 \times \left(1 - \frac{150 + 30 - 60\sqrt{5}}{24^2}\right) = 4 \times \left(1 - \frac{180 - 60\sqrt{5}}{24^2}\right) = 4 \times \left(1 - \frac{15 - 5\sqrt{5}}{2 \times 24}\right) = \\ &= 4 \times \frac{48 - 15 + 5\sqrt{5}}{48} = \boxed{\frac{33 + 5\sqrt{5}}{12}} \end{aligned}$$

cuyos valores, sustituidos en [8], nos dará finalmente:

Page No.	Date	Page No.
----------	------	----------

The given function is $y = \frac{1}{x^2} + \frac{1}{x^3}$. We have to find the derivative of the function with respect to x .

(1) $y = \frac{1}{x^2} + \frac{1}{x^3}$

On differentiating both sides with respect to x , we get

$$\frac{dy}{dx} = \frac{d}{dx} \left(\frac{1}{x^2} + \frac{1}{x^3} \right)$$

$$= \frac{d}{dx} \left(x^{-2} + x^{-3} \right)$$

$$= \frac{d}{dx} x^{-2} + \frac{d}{dx} x^{-3}$$

$$= -2x^{-3} - 3x^{-4}$$

$$= -\frac{2}{x^3} - \frac{3}{x^4}$$

$$= -\frac{2x + 3}{x^4}$$

$$= -\frac{2x + 3}{x^4}$$

$$= -\frac{2x + 3}{x^4}$$

Therefore, the derivative of the function is $-\frac{2x + 3}{x^4}$.

$$\begin{aligned}
 m &= \sqrt{\frac{5 - 6 \sqrt{\frac{2}{3}} \cos \varphi}{4(1 - \cos^2 \varphi)}} \cdot l = \sqrt{\frac{5 + \sqrt{5}}{2} : \frac{33 + 5\sqrt{5}}{12}} \cdot l = \sqrt{\frac{6(5 + \sqrt{5})}{33 + 5\sqrt{5}}} \cdot l = \\
 &= \sqrt{\frac{6(5 + \sqrt{5})(33 - 5\sqrt{5})}{33^2 - 125}} \cdot l = \sqrt{\frac{6(165 + 33\sqrt{5} - 25\sqrt{5} - 25)}{964}} \cdot l = \\
 &= \sqrt{\frac{3(140 + 8\sqrt{5})}{482}} \cdot l = \sqrt{\frac{3(70 + 4\sqrt{5})}{241}} \cdot l = \boxed{\sqrt{\frac{6(35 + 2\sqrt{5})}{241}}} \cdot l = \\
 &= 0,99\ 13\ 16\ 18 \dots l
 \end{aligned}$$

Para el caso del dibujo, será: $m = 0,99\ 13\ 16\ 18 \dots \times 14,46 = 14,3\ mm$

Radio "a" de la esfera circunscrita

Aplicando la fórmula general [1] (ver lám. 33)

$$\begin{aligned}
 a &= \frac{l^2}{2\sqrt{l^2 - m^2}} = \frac{l^2}{2\sqrt{l^2 - \left(\sqrt{\frac{6(35 + 2\sqrt{5})}{241}} l\right)^2}} = \frac{1}{2\sqrt{1 - \frac{6(35 + 2\sqrt{5})}{241}}} \cdot l = \\
 &= \frac{1}{2\sqrt{1 - \frac{210 + 12\sqrt{5}}{241}}} \cdot l = \frac{1}{2\sqrt{\frac{241 - 210 - 12\sqrt{5}}{241}}} \cdot l = \frac{1}{2\sqrt{\frac{31 - 12\sqrt{5}}{241}}} \cdot l = \\
 &= \frac{1}{2} \sqrt{\frac{241}{31 - 12\sqrt{5}}} \cdot l = \frac{1}{2} \sqrt{\frac{241(31 + 12\sqrt{5})}{31^2 - 12^2 \cdot 5}} \cdot l = \frac{1}{2} \sqrt{\frac{241(31 + 12\sqrt{5})}{241}} \cdot l = \\
 &= \frac{1}{2} \sqrt{31 + 12\sqrt{5}} \cdot l = \boxed{\frac{\sqrt{31 + 12\sqrt{5}}}{2}} \cdot l = 3,80\ 23\ 94\ 50 \dots l
 \end{aligned}$$

Para el caso del dibujo, será: $a = 55\ mm$ $l = 14,46\ mm$

1. $\frac{1}{x^2} = x^{-2}$ $\frac{d}{dx} x^{-2} = -2x^{-3} = -\frac{2}{x^3}$

2. $\frac{d}{dx} \frac{1}{x^3} = \frac{d}{dx} x^{-3} = -3x^{-4} = -\frac{3}{x^4}$

3. $\frac{d}{dx} \frac{1}{x^4} = \frac{d}{dx} x^{-4} = -4x^{-5} = -\frac{4}{x^5}$

4. $\frac{d}{dx} \frac{1}{x^5} = \frac{d}{dx} x^{-5} = -5x^{-6} = -\frac{5}{x^6}$

5. $\frac{d}{dx} \frac{1}{x^6} = \frac{d}{dx} x^{-6} = -6x^{-7} = -\frac{6}{x^7}$

6. $\frac{d}{dx} \frac{1}{x^7} = \frac{d}{dx} x^{-7} = -7x^{-8} = -\frac{7}{x^8}$

7. $\frac{d}{dx} \frac{1}{x^8} = \frac{d}{dx} x^{-8} = -8x^{-9} = -\frac{8}{x^9}$

8. $\frac{d}{dx} \frac{1}{x^9} = \frac{d}{dx} x^{-9} = -9x^{-10} = -\frac{9}{x^{10}}$

9. $\frac{d}{dx} \frac{1}{x^{10}} = \frac{d}{dx} x^{-10} = -10x^{-11} = -\frac{10}{x^{11}}$

10. $\frac{d}{dx} \frac{1}{x^{11}} = \frac{d}{dx} x^{-11} = -11x^{-12} = -\frac{11}{x^{12}}$

11. $\frac{d}{dx} \frac{1}{x^{12}} = \frac{d}{dx} x^{-12} = -12x^{-13} = -\frac{12}{x^{13}}$

Radio "b" de la esfera tangente a las aristas

Aplicando la fórmula general [3] (ver lám. 33)

$$b = \sqrt{a^2 - \frac{l^2}{4}} = \sqrt{\left(\frac{\sqrt{31+12\sqrt{5}}}{2} l\right)^2 - \frac{l^2}{4}} = \sqrt{\frac{31+12\sqrt{5}}{4} - \frac{1}{4}} \cdot l =$$

$$= \sqrt{\frac{30+12\sqrt{5}}{4}} \cdot l = \frac{\sqrt{30+12\sqrt{5}}}{2} l = 3,76\ 93\ 77\ 13\dots l$$

Para el caso del dibujo, será: $b = 3,76\ 93\ 77\ 13\dots \times 14,46 = 54,5\ \text{mm}$

Radio "d₄" de la circunferencia circunscrita a una cara cuadrada de lado "l".

Se demuestra en Geometría, es

$$d_4 = \frac{\sqrt{2}}{2} l = 0,70\ 71\ 06\ 78\dots l$$

Para el caso del dibujo, será: $d_4 = 0,70\ 71\ 06\ 78\dots \times 14,46 = 10,2\ \text{mm}$

Radio "d₆" de la circunferencia circunscrita a una cara hexagonal de lado "l".

Se demuestra en Geometría, es

$$d_6 = l$$

Radio "d₁₀" de la circunferencia circunscrita a una

Let us consider a function $f(x)$ defined on the interval $[a, b]$.
 We want to find the area under the curve $y = f(x)$ from $x = a$ to $x = b$.
 This area is given by the definite integral $\int_a^b f(x) dx$.
 To evaluate this integral, we use the Fundamental Theorem of Calculus.
 If $F(x)$ is an antiderivative of $f(x)$, then

$$\int_a^b f(x) dx = F(b) - F(a)$$

For example, let $f(x) = x^2$. Then an antiderivative is $F(x) = \frac{x^3}{3}$.
 The area under the curve $y = x^2$ from $x = 1$ to $x = 2$ is

$$\int_1^2 x^2 dx = \left[\frac{x^3}{3} \right]_1^2 = \frac{2^3}{3} - \frac{1^3}{3} = \frac{8}{3} - \frac{1}{3} = \frac{7}{3}$$

Another example: let $f(x) = \sin(x)$. Then an antiderivative is $F(x) = -\cos(x)$.
 The area under the curve $y = \sin(x)$ from $x = 0$ to $x = \pi$ is

$$\int_0^\pi \sin(x) dx = [-\cos(x)]_0^\pi = -\cos(\pi) - (-\cos(0)) = -(-1) - (-1) = 1 + 1 = 2$$

$$\int_a^b f(x) dx = F(b) - F(a)$$

diagonal de lado "l"

Se demuestra en Geometría, es

$$d_{10} = \frac{\sqrt{5} + 1}{2} l = 1,61803399... l$$

Para el caso del dibujo, será: $d_{10} = 1,61803399... \times 14,46 = 23,41$ mm.Radio "C₄" de la esfera tangente a las caras cuadradas de lado "l"

Aplicando la fórmula general [2] (ver lám. 33)

$$C_4 = \sqrt{a^2 - (d_4)^2} = \sqrt{\left(\frac{\sqrt{31+12\sqrt{5}}}{2} l\right)^2 - \left(\frac{\sqrt{2}}{2} l\right)^2} = \sqrt{\frac{31+12\sqrt{5}}{4} - \frac{2}{4}} \cdot l =$$

$$= \sqrt{\frac{29+12\sqrt{5}}{4}} \cdot l = \frac{\sqrt{29+12\sqrt{5}}}{2} l = 3,73606798... l = \frac{2\sqrt{5}+3}{2} l$$

Para el caso del dibujo, será: $C_4 = 3,73606798... \times 14,46 = 54,0$ mmRadio "C₆" de la esfera tangente a las caras hexagonales de lado "l"

Aplicando la fórmula general [2] (ver lám. 33)

$$C_6 = \sqrt{a^2 - (d_6)^2} = \sqrt{\left(\frac{\sqrt{31+12\sqrt{5}}}{2} l\right)^2 - l^2} = \sqrt{\frac{31+12\sqrt{5}}{4} - 1} \cdot l =$$

$$= \sqrt{\frac{27+12\sqrt{5}}{4}} \cdot l = \frac{\sqrt{27+12\sqrt{5}}}{2} \cdot l = \frac{\sqrt{3(9+4\sqrt{5})}}{2} \cdot l = \frac{\sqrt{3} \times \left(\sqrt{\frac{10}{2}} + \sqrt{\frac{8}{2}}\right)}{2} \cdot l$$

* continúa el cálculo en el reverso

$$* \frac{\sqrt{29 + 12\sqrt{5}}}{2} = \frac{\sqrt{\frac{29+11}{2}} + \sqrt{\frac{29-11}{2}}}{2} = \frac{\sqrt{20} + \sqrt{9}}{2} = \frac{2\sqrt{5} + 3}{2}$$

Handwritten text in a cursive script, possibly Urdu or Persian, located at the top of the page. The text is faint and appears to be a title or header.

$$= \frac{\sqrt{3}(\sqrt{5}+2)}{2} \cdot l = \frac{\sqrt{15}+2\sqrt{3}}{2} \cdot l = 3,66\ 85\ 42\ 49 \dots l$$

Para el caso del dibujo, sea: $C_6 = 3,66\ 85\ 42\ 49 \dots \times 14,46 = 53,0\text{ mm}$

Radio "C₁₀" de la esfera tangente a las caras decagonales de lado "l"

Aplicando la fórmula general [2] (ver lám. 33)

$$C_{10} = \sqrt{a^2 - (d_{10})^2} = \sqrt{\left(\frac{\sqrt{31+12\sqrt{5}}}{2} l\right)^2 - \left(\frac{\sqrt{5}+1}{2} l\right)^2} =$$

$$= \sqrt{\frac{31+12\sqrt{5}}{4} - \frac{6+2\sqrt{5}}{4}} \cdot l = \sqrt{\frac{25+10\sqrt{5}}{4}} \cdot l = \frac{\sqrt{25+10\sqrt{5}}}{2} \cdot l =$$

$$= 3,44\ 09\ 54\ 80 \dots l$$

Para el caso del dibujo, sea: $C_{10} = 3,44\ 09\ 54\ 80 \dots \times 14,46 = 49,8\text{ mm}$

Ángulo rectilíneo "α₄" del diedro formado por una cara cuadrada, con el plano diametral del arquimedianos que pasa por una arista de aquella.

Se obtiene, en función de su tangente, por la fórmula general [5] (ver lám. 33)

$$\tan \alpha_4 = \frac{2 C_4}{\sqrt{4(d_4)^2 - l^2}} = \frac{2 \times \frac{\sqrt{29+12\sqrt{5}}}{2} l}{\sqrt{4\left(\frac{\sqrt{2}}{2} l\right)^2 - l^2}} = \frac{\sqrt{29+12\sqrt{5}}}{\sqrt{4 \times \frac{1}{2} - 1}} = \sqrt{29+12\sqrt{5}}$$

* o ver mayor simplificación en el reverso

--	--	--	--

The first part of the paper is devoted to the study of the
 properties of the function $f(x)$ defined by the
 equation

$$f(x) = \frac{1}{2} \left(f\left(\frac{x}{2}\right) + f\left(\frac{x+1}{2}\right) \right)$$

and to the study of the function $g(x)$ defined by the
 equation

$$g(x) = \frac{1}{2} \left(g\left(\frac{x}{2}\right) + g\left(\frac{x+1}{2}\right) \right)$$

$$= \frac{1}{2} \left(\frac{1}{2} \left(g\left(\frac{x}{4}\right) + g\left(\frac{x+1}{4}\right) \right) + \frac{1}{2} \left(g\left(\frac{x+1}{4}\right) + g\left(\frac{x+2}{4}\right) \right) \right)$$

and to the study of the function $h(x)$ defined by the
 equation

$$h(x) = \frac{1}{2} \left(h\left(\frac{x}{2}\right) + h\left(\frac{x+1}{2}\right) \right)$$

and to the study of the function $i(x)$ defined by the
 equation

$$i(x) = \frac{1}{2} \left(i\left(\frac{x}{2}\right) + i\left(\frac{x+1}{2}\right) \right)$$

and to the study of the function $j(x)$ defined by the
 equation

$$j(x) = \frac{1}{2} \left(j\left(\frac{x}{2}\right) + j\left(\frac{x+1}{2}\right) \right)$$

*

$$\sqrt{29 + 12\sqrt{5}} = \sqrt{\frac{29+11}{2}} + \sqrt{\frac{29-11}{2}} = \sqrt{20} + \sqrt{9} =$$

$$= \boxed{2\sqrt{5} + 3}$$

Handwritten text, possibly a title or header, mostly illegible due to fading.

1894

$$= 7, 47 \ 21 \ 35 \ 96 \dots$$

$$\lg \operatorname{tg} \alpha_4 = 0, 87 \ 34 \ 44 \ 2$$

$$\alpha_4 = 82^\circ \ 22' \ 38,5''$$

Ángulo rectilíneo " α_6 " del diedro formado por una cara exagonal, con el plano diametral del arquimedianos que pasa por una arista de aquella.

Se obtiene, en función de su tangente, aplicando la fórmula general [5] (ver lám. 33).

$$\boxed{\lg \alpha_6} = \frac{2 C_6}{\sqrt{4 (d_6)^2 - l^2}} = \frac{2 \times \frac{\sqrt{15} + 2\sqrt{3}}{2} l}{\sqrt{4 l^2 - l^2}} = \frac{\sqrt{15} + 2\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{45} + 6}{3} =$$

$$= \frac{3\sqrt{5} + 6}{3} = \sqrt{5} + 2 = 4, 23 \ 60 \ 67 \ 98$$

$$\lg \operatorname{tg} \alpha_6 = 0, 62 \ 69 \ 62 \ 9$$

$$\alpha_6 = 76^\circ \ 43' \ 2,9''$$

Ángulo rectilíneo " α_{10} " del diedro formado por una cara decagonal, con el plano diametral del arquimedianos que pasa por una arista de aquella.

Se obtiene, en función de su tangente, aplicando la fórmula general [5] (ver lám. 33)

$$\boxed{\lg \alpha_{10}} = \frac{2 C_{10}}{\sqrt{4 (d_{10})^2 - l^2}} = \frac{2 \times \frac{\sqrt{35} + 10\sqrt{5}}{2} l}{\sqrt{4 \times \left(\frac{\sqrt{5} + 1}{2} l \right)^2 - l^2}} = \frac{\sqrt{25 + 10\sqrt{5}}}{\sqrt{4 \times \frac{6 + 2\sqrt{5}}{4} - 1}} =$$

The ...

...

...

...

...

The ...

...

...

...

$$= \frac{\sqrt{25+10\sqrt{5}}}{\sqrt{6+2\sqrt{5}-1}} = \sqrt{\frac{25+10\sqrt{5}}{5+2\sqrt{5}}} = \sqrt{\frac{(25+10\sqrt{5})(5-2\sqrt{5})}{25-20}} = \sqrt{\frac{125+50\sqrt{5}-50\sqrt{5}-100}{5}} =$$

$$= \sqrt{\frac{25}{5}} = \sqrt{5} = 2, 23 \ 60 \ 67 \ 98''$$

$$t_g \ t_g \ \alpha_{10} = 0, 34 \ 94 \ 85 \ 0$$

$$\alpha_{10} = 65^\circ \ 54' \ 18,6''$$

Angulo rectilíneo " φ_{4-6} " del diedro formado por una cara cuadrada y otra exagonal regular

Aplicando la fórmula general [4] (ver lám. 33)

$$\boxed{\varphi_{4-6}} = \alpha_4 + \alpha_6 = 82^\circ \ 22' \ 38,5'' + 76^\circ \ 43' \ 2,9'' =$$

$$= \boxed{159^\circ \ 5' \ 41,4''}$$

También puede obtenerse directamente este valor:

$$\boxed{t_g \ \varphi_{4-6}} = t_g (\alpha_4 + \alpha_6) = \frac{t_g \alpha_4 + t_g \alpha_6}{1 - t_g \alpha_4 t_g \alpha_6} = \frac{\sqrt{29+12\sqrt{5}} + (\sqrt{5}+2)}{1 - \sqrt{29+12\sqrt{5}} \times (\sqrt{5}+2)} =$$

$$= \frac{\sqrt{29+12\sqrt{5}} + (\sqrt{5}+2)}{1 - \sqrt{(29+12\sqrt{5})(\sqrt{5}+2)^2}} = \frac{\sqrt{29+12\sqrt{5}} + (\sqrt{5}+2)}{1 - \sqrt{(29+12\sqrt{5})(9+4\sqrt{5})}} = \frac{\sqrt{29+12\sqrt{5}} + (\sqrt{5}+2)}{1 - \sqrt{261+108\sqrt{5}+116\sqrt{5}+240}} =$$

$$= \frac{\sqrt{29+12\sqrt{5}} + (\sqrt{5}+2)}{1 - \sqrt{501+224\sqrt{5}}} = \frac{\sqrt{29+12\sqrt{5}} + (\sqrt{5}+2)}{1 - \left(\sqrt{\frac{501+11}{2}} + \sqrt{\frac{501-11}{2}} \right)} = \frac{\sqrt{29+12\sqrt{5}} + (\sqrt{5}+2)}{1 - \sqrt{256} - \sqrt{245}} =$$

$$= \frac{\sqrt{29+12\sqrt{5}} + (\sqrt{5}+2)}{1 - 16 - 7\sqrt{5}} = - \frac{\sqrt{29+12\sqrt{5}} + (\sqrt{5}+2)}{7\sqrt{5} + 15} = - \frac{[\sqrt{29+12\sqrt{5}} + (\sqrt{5}+2)] \cdot (7\sqrt{5}-15)}{7^2 \cdot 5 - 15^2} =$$

--	--	--

1. $\frac{1}{2} + \frac{1}{3} = \frac{3}{6} + \frac{2}{6} = \frac{5}{6}$

2. $\frac{1}{4} - \frac{1}{8} = \frac{2}{8} - \frac{1}{8} = \frac{1}{8}$

3. $\frac{3}{5} \times \frac{2}{3} = \frac{3 \times 2}{5 \times 3} = \frac{6}{15} = \frac{2}{5}$

4. $\frac{2}{3} \div \frac{4}{5} = \frac{2}{3} \times \frac{5}{4} = \frac{2 \times 5}{3 \times 4} = \frac{10}{12} = \frac{5}{6}$

5. $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} = \frac{4}{8} + \frac{2}{8} + \frac{1}{8} = \frac{7}{8}$

6. $\frac{3}{4} - \frac{1}{2} = \frac{3}{4} - \frac{2}{4} = \frac{1}{4}$

7. $\frac{1}{3} \times \frac{3}{4} = \frac{1 \times 3}{3 \times 4} = \frac{3}{12} = \frac{1}{4}$

8. $\frac{2}{5} \div \frac{3}{10} = \frac{2}{5} \times \frac{10}{3} = \frac{2 \times 10}{5 \times 3} = \frac{20}{15} = \frac{4}{3}$

9. $\frac{1}{2} + \frac{1}{3} + \frac{1}{6} = \frac{2}{6} + \frac{2}{6} + \frac{1}{6} = \frac{5}{6}$

10. $\frac{3}{4} - \frac{1}{2} = \frac{3}{4} - \frac{2}{4} = \frac{1}{4}$

11. $\frac{1}{2} \times \frac{3}{4} = \frac{1 \times 3}{2 \times 4} = \frac{3}{8}$

--	--	--

$$= - \frac{(7\sqrt{5} - 15) \cdot \sqrt{29 + 12\sqrt{5}} + (\sqrt{5} + 2)(7\sqrt{5} - 15)}{20} =$$

$$= - \frac{\sqrt{(29 + 12\sqrt{5})(7\sqrt{5} - 15)^2} + (35 + 14\sqrt{5} - 15\sqrt{5} - 30)}{20} =$$

$$= - \frac{\sqrt{(29 + 12\sqrt{5})(245 + 225 - 210\sqrt{5})} + (5 - \sqrt{5})}{20} = - \frac{\sqrt{(29 + 12\sqrt{5})(470 - 210\sqrt{5})} + (5 - \sqrt{5})}{20} =$$

$$= - \frac{\sqrt{10(29 + 12\sqrt{5})(47 - 21\sqrt{5})} + (5 - \sqrt{5})}{20} = - \frac{\sqrt{10(1363 + 564\sqrt{5} - 609\sqrt{5} - 1260)} + (5 - \sqrt{5})}{20} =$$

$$= - \frac{\sqrt{10(103 - 45\sqrt{5})} + (5 - \sqrt{5})}{20} = - \frac{\sqrt{10} \cdot \left(\sqrt{\frac{103 + 22}{2}} - \sqrt{\frac{103 - 22}{2}} \right) + (5 - \sqrt{5})}{20} =$$

$$= - \frac{\sqrt{10} \left(\sqrt{\frac{125}{2}} - \sqrt{\frac{81}{2}} \right) + (5 - \sqrt{5})}{20} = - \frac{\sqrt{\frac{1250}{2}} - \sqrt{\frac{810}{2}} + 5 - \sqrt{5}}{20} =$$

$$= - \frac{\sqrt{625} - \sqrt{405} + 5 - \sqrt{5}}{20} = - \frac{25 - 9\sqrt{5} + 5 - \sqrt{5}}{20} = - \frac{30 - 10\sqrt{5}}{20} =$$

$$= - \frac{3 - \sqrt{5}}{2} = - 0,38196601..$$

Y haciendo $\alpha_0 = \pi - \varphi_{4-6}$, sea' $\frac{1}{2} \alpha_0 = - \frac{1}{2} \varphi_{4-6} =$

$$= - \left(- \frac{3 - \sqrt{5}}{2} \right) = \frac{3 - \sqrt{5}}{2} = 0,38196601..$$

$$\frac{1}{2} \frac{1}{2} \alpha_0 = \bar{7},5820248$$

$$\alpha_0 = 20^\circ 54' 18,6''$$

Y finalmente $\varphi_{4-6} = 180^\circ - 20^\circ 54' 18,6'' = 159^\circ 5' 41,4''$

* Ver simplificación en el reverso

... ..
... ..

... ..
... ..

... ..
... ..

... ..
... ..

... ..
... ..

... ..
... ..

*

El valor obtenido se hubiese hallado más fácilmente utilizando el más simplificado de

$$\boxed{\operatorname{tg} \alpha_4} = \sqrt{29 + 12\sqrt{5}} = \boxed{2\sqrt{5} + 3}$$

que se detalla al dorso de la hoja 9.

Los cálculos, menos laboriosos, hubiesen sido:

$$\boxed{\operatorname{tg} \varphi_{4-6}} = \operatorname{tg} (\alpha_4 + \alpha_6) = \frac{\operatorname{tg} \alpha_4 + \operatorname{tg} \alpha_6}{1 - \operatorname{tg} \alpha_4 \operatorname{tg} \alpha_6} = \frac{(2\sqrt{5}+3) + (\sqrt{5}+2)}{1 - (2\sqrt{5}+3)(\sqrt{5}+2)} =$$

$$= \frac{3\sqrt{5} + 5}{1 - (10 + 3\sqrt{5} + 4\sqrt{5} + 6)} = \frac{3\sqrt{5} + 5}{1 - (16 + 7\sqrt{5})} = \frac{3\sqrt{5} + 5}{-15 - 7\sqrt{5}} = - \frac{3\sqrt{5} + 5}{7\sqrt{5} + 15} =$$

$$= - \frac{(3\sqrt{5} + 5)(7\sqrt{5} - 15)}{49 \times 5 - 15^2} = - \frac{21 \times 5 + 35\sqrt{5} - 45\sqrt{5} - 75}{20} = - \frac{30 - 10\sqrt{5}}{20} =$$

$$= - \frac{3 - \sqrt{5}}{2}$$

The following table is a summary of the results of the experiments conducted on the effect of the temperature of the water on the rate of the reaction.

It is seen from the table that the rate of the reaction increases with the temperature of the water. This is due to the fact that the rate of the reaction is proportional to the rate constant, which is a function of the temperature. The rate constant is given by the Arrhenius equation:

$$k = A e^{-\frac{E_a}{RT}}$$

where k is the rate constant, A is the pre-exponential factor, E_a is the activation energy, R is the gas constant, and T is the absolute temperature.

$$\ln k = \ln A - \frac{E_a}{RT}$$

Thus, a plot of $\ln k$ versus $1/T$ should give a straight line with a slope of $-E_a/R$.

(24)

valor coincidente con el calculado anteriormente.

Ángulo rectilíneo " φ_{4-8} " del diedro formado por una cara cuadrada y otra octogonal

Aplicando la fórmula general [4] (ver lám. 33)

$$\boxed{\varphi_{4-10}} = \alpha_4 + \alpha_{10} = 82^\circ 22' 38,5'' + 65^\circ 54' 18,6'' =$$

$$= \boxed{148^\circ 16' 57,7''}$$

Puede obtenerse directamente, así:

$$\boxed{t_g \varphi_{4-10}} = t_g (\alpha_4 + \alpha_{10}) = \frac{t_g \alpha_4 + t_g \alpha_{10}}{1 - t_g \alpha_4 \cdot t_g \alpha_{10}} = \frac{\frac{\sqrt{29+12\sqrt{5}}}{1-\sqrt{5(29+12\sqrt{5})}} + \frac{\sqrt{5}}{1-\sqrt{5(29+12\sqrt{5})}}}{1 - \frac{\sqrt{29+12\sqrt{5}}}{1-\sqrt{5(29+12\sqrt{5})}} \cdot \frac{\sqrt{5}}{1-\sqrt{5(29+12\sqrt{5})}}} =$$

$$= \frac{\frac{\sqrt{29+12\sqrt{5}} + \sqrt{5}}{1-\sqrt{5(29+12\sqrt{5})}}}{1 - \frac{5(29+12\sqrt{5})}{(1-\sqrt{5(29+12\sqrt{5})})^2}} = \frac{(\sqrt{29+12\sqrt{5}} + \sqrt{5})(1 + \sqrt{5(29+12\sqrt{5})})}{1 - 5(29+12\sqrt{5})} =$$

$$= \frac{\sqrt{29+12\sqrt{5}} + \sqrt{5} + \sqrt{5}(29+12\sqrt{5}) + 5\sqrt{29+12\sqrt{5}}}{1 - 145 - 60\sqrt{5}} = \frac{6\sqrt{29+12\sqrt{5}} + \sqrt{5}(30+12\sqrt{5})}{-144 - 60\sqrt{5}} =$$

$$= - \frac{\sqrt{29+12\sqrt{5}} + \sqrt{5}(5+3\sqrt{5})}{24 + 10\sqrt{5}} = - \frac{\sqrt{29+12\sqrt{5}} + (5\sqrt{5} + 10)}{2(12 + 5\sqrt{5})} =$$

$$= - \frac{[\sqrt{29+12\sqrt{5}} + 5(\sqrt{5}+2)] \cdot (12-5\sqrt{5})}{2(12^2 - 5^2 \cdot 5)} = - \frac{\sqrt{(29+12\sqrt{5})(12-5\sqrt{5})^2} + 5(\sqrt{5}+2)(12-5\sqrt{5})}{2 \times 19} =$$

$$= - \frac{\sqrt{(29+12\sqrt{5})(144 + 125 - 120\sqrt{5})} + 5(12\sqrt{5} + 24 - 25 - 10\sqrt{5})}{2 \times 19} =$$

$$= - \frac{\sqrt{(29+12\sqrt{5})(269 - 120\sqrt{5})} + 5(2\sqrt{5} - 1)}{2 \times 19} =$$

The following are the questions asked in the examination.
 Question 1: A man starts walking from his house at 8:00 AM.
 He walks at a speed of 5 km/hr. He reaches his office at 10:00 AM.
 How far is his office from his house?

Solution:

$$\text{Distance} = \text{Speed} \times \text{Time}$$

$$= 5 \text{ km/hr} \times 2 \text{ hr} = 10 \text{ km}$$

Question 2: A train starts from station A at 10:00 AM and reaches station B at 12:00 PM. The distance between A and B is 100 km. What is the speed of the train?

Solution:

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

$$= \frac{100 \text{ km}}{2 \text{ hr}} = 50 \text{ km/hr}$$

$$= - \frac{\sqrt{29 \times 269 + 12 \times 269 \sqrt{5} - 29 \times 120 \sqrt{5} - 60 \times 120} + 5(2\sqrt{5} - 1)}{2 \times 19} =$$

$$= - \frac{\sqrt{601 - 252 \sqrt{5}} + 5(2\sqrt{5} - 1)}{2 \times 19} = - \frac{\sqrt{\frac{601 + 209}{2}} - \sqrt{\frac{601 - 209}{2}} + 10\sqrt{5} - 5}{2 \times 19} =$$

$$= - \frac{\sqrt{405} - \sqrt{196} + 10\sqrt{5} - 5}{2 \times 19} = - \frac{9\sqrt{5} - 14 + 10\sqrt{5} - 5}{2 \times 19} = - \frac{19\sqrt{5} - 19}{2 \times 19} =$$

$$= - \frac{\sqrt{5} - 1}{2} = - 0,61803399\dots$$

y haciendo $\alpha_0 = \pi - \varphi_{4-10}$, será: $\lg \alpha_0 = - \lg \varphi_{4-10} =$

$$= - \left(- \frac{\sqrt{5} - 1}{2} \right) = \frac{\sqrt{5} - 1}{2} = 0,61803399\dots$$

$$\lg \lg \alpha_0 = 7,7910124$$

$$\alpha_0 = 31^\circ 43' 2,9''$$

y finalmente: $\varphi_{4-10} = 180^\circ - 31^\circ 43' 2,9'' = 148^\circ 16' 57,7''$

valor coincidente con el calculado anteriormente.

Ángulo rectilíneo " φ_{6-10} " del diedro formado por una cara exagonal y otra octogonal.

Aplicando la fórmula general [4] (ver lám. 33)

$$\varphi_{6-10} = \alpha_6 + \alpha_{10} = 76^\circ 43' 2,9'' + 65^\circ 54' 18,6'' =$$

$$= 142^\circ 37' 21,5''$$

* Ver simplificación en el reverso

* El valor obtenido se hubiese hallado más fácilmente, utilizando el más simplificado de

$$\operatorname{tg} \alpha_4 = \sqrt{29 + 12\sqrt{5}} = 2\sqrt{5} + 3$$

que se detalla al dorso de la hoja 9.

Los cálculos, menos laboriosos, hubiesen sido:

$$\begin{aligned} \boxed{\operatorname{tg} \varphi_{4-10}} &= \operatorname{tg} (\alpha_4 + \alpha_{10}) = \frac{\operatorname{tg} \alpha_4 + \operatorname{tg} \alpha_{10}}{1 - \operatorname{tg} \alpha_4 \operatorname{tg} \alpha_{10}} = \frac{(2\sqrt{5}+3) + \sqrt{5}}{1 - (2\sqrt{5}+3)(\sqrt{5})} = \\ &= \frac{3\sqrt{5} + 3}{1 - (10 + 3\sqrt{5})} = \frac{3\sqrt{5} + 3}{-9 - 3\sqrt{5}} = - \frac{3\sqrt{5} + 3}{9 + 3\sqrt{5}} = - \frac{\sqrt{5} + 1}{3 + \sqrt{5}} = - \frac{(\sqrt{5}+1)(3-\sqrt{5})}{4} = \\ &= - \frac{3\sqrt{5} + 3 - 5 - \sqrt{5}}{4} = - \frac{2\sqrt{5} - 2}{4} = \boxed{- \frac{\sqrt{5} - 1}{2}} \end{aligned}$$

Puede obtenerse directamente, así:

$$\begin{aligned} \boxed{\tan \varphi_{6-10}} &= \tan (\alpha_6 + \alpha_{10}) = \frac{\tan \alpha_6 + \tan \alpha_{10}}{1 - \tan \alpha_6 \times \tan \alpha_{10}} = \frac{(\sqrt{5}+2) + \sqrt{5}}{1 - (\sqrt{5}+2)\sqrt{5}} = \\ &= \frac{2(\sqrt{5}+1)}{1 - (5+2\sqrt{5})} = \frac{2(\sqrt{5}+1)}{-4-2\sqrt{5}} = -\frac{2(\sqrt{5}+1)}{2(\sqrt{5}+2)} = -\frac{(\sqrt{5}+1)(\sqrt{5}-2)}{1} = \\ &= -(5 + \sqrt{5} - 2\sqrt{5} - 2) = \boxed{-(3 - \sqrt{5})} = -0,76393202... \end{aligned}$$

haciendo $\alpha_0 = \pi - \varphi_{6-10}$, sea' $\tan \alpha_0 = -\tan \varphi_{6-10} =$

$$= -(- (3 - \sqrt{5})) = 3 - \sqrt{5} = 0,76393202...$$

$$\tan \alpha_0 = 0,76393202 \quad \alpha_0 = 37^\circ 22' 38,5''$$

7 finalmente $\boxed{\varphi_{6-10}} = 180^\circ - 37^\circ 22' 38,5'' = \boxed{142^\circ 37' 21,5''}$

valor coincidente con el ya calculado anteriormente.

Área lateral "S" del arquimedianos

Se compone de la suma de 30 caras cuadradas, 30 hexagonales y 12 decagonales

La apotema de la cara hexagonal, será: (ver lám. 42, h 9)

$$\text{apotema } P_6 = \frac{\sqrt{3}}{2} l$$

La apotema de la cara decagonal, será: (ver lám. 41, h 12)

$$\frac{\sqrt{5+2\sqrt{5}}}{2} l$$

y el área lateral S

$$\begin{aligned} S &= 30 l^2 + 20 \times \frac{6}{2} \times \frac{\sqrt{3}}{2} l^2 + 12 \times \frac{10}{2} \times \frac{\sqrt{5+2\sqrt{5}}}{2} l^2 = \\ &= (30 + 30\sqrt{3} + 30\sqrt{5+2\sqrt{5}}) l^2 = \boxed{30(1 + \sqrt{3} + \sqrt{5+2\sqrt{5}}) l^2} = \\ &= 174,29203020 \dots l^2 \end{aligned}$$

Volumen "V" del arquimedianos

Se compone de la suma de 30 pirámides regulares de base cuadrada y altura " C_4 "; de 20 pirámides hexagonales de altura " C_6 " y de 12 dodecagonales de altura " C_{10} ". Su volumen será pues:

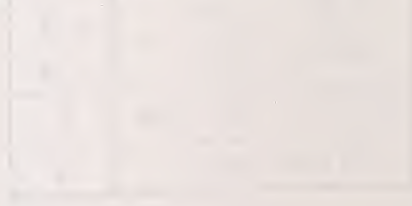
$$\begin{aligned} V &= 30 l^2 \times \frac{2\sqrt{5}+3}{2 \times 3} l + 30\sqrt{3} l^2 \times \frac{\sqrt{5}+2\sqrt{3}}{2 \times 3} l + 30\sqrt{5+2\sqrt{5}} l^2 \times \frac{\sqrt{25+10\sqrt{5}}}{2 \times 3} l = \\ &= \left[5(2\sqrt{5}+3) + 5\sqrt{3}(\sqrt{5}+2\sqrt{3}) + 5\sqrt{5+2\sqrt{5}} \times \sqrt{25+10\sqrt{5}} \right] l^3 = \\ &= \left[10\sqrt{5} + 15 + 5(\sqrt{45} + 6) + 5\sqrt{(5+2\sqrt{5})(25+10\sqrt{5})} \right] l^3 = \\ &= \left(10\sqrt{5} + 15 + 5 \times 3\sqrt{5} + 30 + 5\sqrt{125 + 50\sqrt{5} + 50\sqrt{5} + 100} \right) l^3 = \\ &= (25\sqrt{5} + 45 + 5\sqrt{225 + 100\sqrt{5}}) l^3 = (25\sqrt{5} + 45 + 5\sqrt{25(9 + 4\sqrt{5})}) l^3 = \end{aligned}$$

$$\begin{aligned}
 &= (25\sqrt{5} + 45 + 25\sqrt{9+4\sqrt{5}}) l^3 = \left[25\sqrt{5} + 45 + 25 \left(\sqrt{\frac{10}{2}} + \sqrt{\frac{6}{2}} \right) \right] l^3 = \\
 &= (25\sqrt{5} + 45 + 25\sqrt{5} + 50) l^3 = (50\sqrt{5} + 95) l^3 = \boxed{5(19 + 10\sqrt{5}) l^3} = \\
 &= 206,80339888... l^3
 \end{aligned}$$

FIGURA CORPÓREA

Se obtiene por acoplamiento de 30 cuadrados de lado $l = 14,5 \text{ mm}$; de 20 exágonos y 12 decágonos, también regulares y de igual lado. El acoplamiento deberá hacerse de forma que en cada vértice concurren un cuadrado, un exágono y un decágono

En el cuadro sinóptico que damos a continuación, resumimos los resultados analíticos obtenidos anteriormente.



1. I am writing to you because I am interested in
the information you have provided me with.

2. I am writing to you because I am interested in
the information you have provided me with.

3. I am writing to you because I am interested in
the information you have provided me with.

4. I am writing to you because I am interested in
the information you have provided me with.

5. I am writing to you because I am interested in
the information you have provided me with.

6. I am writing to you because I am interested in
the information you have provided me with.

CUADRO SINÓPTICO

Magnitud	Valor exacto	Valor decimal aproximado
a	$\frac{\sqrt{37} + 12\sqrt{5}}{2} \ell$	3, 80 23 95... ℓ
b	$\frac{\sqrt{30} + 12\sqrt{5}}{2} \ell$	3, 76 93 77... ℓ
c_4	$\frac{2\sqrt{5} + 3}{2} \ell$	3, 73 60 68... ℓ
c_6	$\frac{\sqrt{15} + 2\sqrt{3}}{2} \ell$	3, 66 85 42... ℓ
c_{10}	$\frac{\sqrt{25} + 10\sqrt{5}}{2} \ell$	3, 44 09 55... ℓ
d_4	$\frac{\sqrt{2}}{2} \ell$	0, 70 71 07... ℓ
d_6	1ℓ	1, 00 00 00... ℓ
d_{10}	$\frac{\sqrt{5} + 1}{2} \ell$	1, 61 80 34... ℓ
m	$\sqrt{\frac{6(35 + 2\sqrt{5})}{241}} \ell$	0, 99 13 16... ℓ
α_4	$\operatorname{tg} \alpha_4 = 2\sqrt{5} + 3$	$\operatorname{tg} \alpha_4 = 7, 47 21 35...$ $\alpha_4 = 82^\circ 22' 38,5''$
α_6	$\operatorname{tg} \alpha_6 = \sqrt{5} + 2$	$\operatorname{tg} \alpha_6 = 4, 23 60 68...$ $\alpha_6 = 76^\circ 43' 2,9''$
α_{10}	$\operatorname{tg} \alpha_{10} = \sqrt{5}$	$\operatorname{tg} \alpha_{10} = 2, 23 60 68...$ $\alpha_{10} = 65^\circ 54' 18,6''$
φ_{4-6}	$\operatorname{tg} \varphi_{4-6} = -\frac{3 - \sqrt{5}}{2}$	$\operatorname{tg} \varphi_{4-6} = -0, 38 19 66...$ $\varphi_{4-6} = 159^\circ 5' 41,4''$
φ_{4-10}	$\operatorname{tg} \varphi_{4-10} = -\frac{\sqrt{5} - 1}{2}$	$\operatorname{tg} \varphi_{4-10} = -0, 61 80 34...$ $\varphi_{4-10} = 148^\circ 16' 57,1''$
φ_{6-10}	$\operatorname{tg} \varphi_{6-10} = -(3 - \sqrt{5})$	$\operatorname{tg} \varphi_{6-10} = -0, 76 39 32$ $\varphi_{6-10} = 142^\circ 37' 21,5''$
S	$30(1 + \sqrt{3} + \sqrt{5} + 2\sqrt{5}) \ell^2$	174, 29 20 30... ℓ^2
V	$5(19 + 10\sqrt{5}) \ell^3$	206, 80 33 99... ℓ^3

Mathematics

Problem	Solution	Answer
1. Find the sum of 12 and 18.	$12 + 18 = 30$	30
2. Find the difference of 25 and 10.	$25 - 10 = 15$	15
3. Find the product of 4 and 6.	$4 \times 6 = 24$	24
4. Find the quotient of 24 and 4.	$24 \div 4 = 6$	6
5. Find the sum of 15 and 20.	$15 + 20 = 35$	35
6. Find the difference of 30 and 12.	$30 - 12 = 18$	18
7. Find the product of 5 and 8.	$5 \times 8 = 40$	40
8. Find the quotient of 36 and 6.	$36 \div 6 = 6$	6
9. Find the sum of 22 and 14.	$22 + 14 = 36$	36
10. Find the difference of 40 and 15.	$40 - 15 = 25$	25
11. Find the product of 7 and 9.	$7 \times 9 = 63$	63
12. Find the quotient of 48 and 8.	$48 \div 8 = 6$	6
13. Find the sum of 28 and 16.	$28 + 16 = 44$	44
14. Find the difference of 50 and 20.	$50 - 20 = 30$	30
15. Find the product of 10 and 12.	$10 \times 12 = 120$	120
16. Find the quotient of 120 and 10.	$120 \div 10 = 12$	12
17. Find the sum of 33 and 17.	$33 + 17 = 50$	50
18. Find the difference of 60 and 25.	$60 - 25 = 35$	35
19. Find the product of 15 and 14.	$15 \times 14 = 210$	210
20. Find the quotient of 210 and 15.	$210 \div 15 = 14$	14

PROCESO GRÁFICO-ANALÍTICO

Después del cálculo de las magnitudes principales, vamos a proceder en la lámina 44, a la representación gráfica del Arquimedianos XII.

Para su trazado nos valdremos de cotas calculadas por las fórmulas anteriores, de procesos gráficos y de cotas complementarias, cuyo cálculo efectuaremos posteriormente. Todas las magnitudes las obtendremos en función del lado " l_{XII} " del arquimedianos, cuya longitud es de 14,46 mm.

Con este objeto, calcularemos previamente las siguientes magnitudes:

$$\begin{aligned}
 l_{XII} &= \text{Dato del ejercicio} &= 14,5 \text{ mm} \\
 a &= 3,80 \ 23 \ 95... \times 14,46 &= 55,0 \text{ mm} \\
 b &= 3,76 \ 93 \ 77... \times 14,46 &= 54,5 \text{ mm} \\
 c_h &= 3,73 \ 60 \ 68... \times 14,46 &= 54,0 \text{ mm} \\
 c_6 &= 3,66 \ 85 \ 42... \times 14,46 &= 53,0 \text{ mm} \\
 c_{10} &= 3,44 \ 09 \ 55... \times 14,46 &= 49,8 \text{ mm} \\
 d_h &= 0,70 \ 71 \ 07... \times 14,46 &= 10,2 \text{ mm} \\
 d_6 &= 1,00 \ 00 \ 00... \times 14,46 &= 14,5 \text{ mm} \\
 d_{10} &= 1,61 \ 80 \ 34... \times 14,46 &= 23,4 \text{ mm}
 \end{aligned}$$

Antes de proceder al trazado gráfico, observemos en la lámina 44 que la proyección del arquimedianos en el plano II, presenta una forma muy regular, debido a la posición elegida en su representación. Esta regularidad nos permite el trazado previo y directo de dicha pro-

THE HISTORY OF THE

The history of the world is a long and varied one, and it is not possible to give a full account of it in a single volume. The history of the world is a story of the human race, and it is a story that is constantly changing. The history of the world is a story of the human race, and it is a story that is constantly changing. The history of the world is a story of the human race, and it is a story that is constantly changing.

The history of the world is a story of the human race, and it is a story that is constantly changing. The history of the world is a story of the human race, and it is a story that is constantly changing. The history of the world is a story of the human race, and it is a story that is constantly changing. The history of the world is a story of the human race, and it is a story that is constantly changing.

The history of the world is a story of the human race, and it is a story that is constantly changing. The history of the world is a story of the human race, and it is a story that is constantly changing. The history of the world is a story of the human race, and it is a story that is constantly changing. The history of the world is a story of the human race, and it is a story that is constantly changing.

gección, la cual nos facilitará la obtención de las proyecciones I y III, según veremos a continuación.

Teniendo presente lo expuesto, el orden de operaciones del trazado gráfico (lámin. 44), es el siguiente:

- 1° Situar el centro O, de coordenadas O (72, 72, 85) mm.
- 2° Dibujar en I, II y III las proyecciones de la esfera circunscrita, de radio 55 mm.
- 3° Comenzar el trazado de la proyección II, dividiendo previamente con gran exactitud y a partir de A, la circunferencia proyección de la esfera inscrita, en 10 partes iguales. (Ver nota al dorso)
- 4° Unir los puntos de división con el centro
- 5° Trazar paralelos a ambos lados de los radios anteriores a distancias iguales a la mitad del lado "b_{XII}" del arquimediaco. Sobre estas rectas están las proyecciones de "todos" los vértices del arquimediaco. Para determinarlos, deberán trazarse circunferencias concéntricas en la exterior y sucesivamente con los siguientes radios

- a) Radio " d_{10} "
- b) Radio " r_1 "
- c) Radio " r_2 "
- d) Radio " r_3 "

The first part of the report is devoted to a description of the
 experimental apparatus and the method of observation. The
 results of the experiments are then given, and a discussion
 of the results is presented. The report concludes with a
 summary of the work and a list of references.

The second part of the report is devoted to a description of the
 experimental apparatus and the method of observation. The
 results of the experiments are then given, and a discussion
 of the results is presented. The report concludes with a
 summary of the work and a list of references.

The third part of the report is devoted to a description of the
 experimental apparatus and the method of observation. The
 results of the experiments are then given, and a discussion
 of the results is presented. The report concludes with a
 summary of the work and a list of references.

The fourth part of the report is devoted to a description of the
 experimental apparatus and the method of observation. The
 results of the experiments are then given, and a discussion
 of the results is presented. The report concludes with a
 summary of the work and a list of references.

The fifth part of the report is devoted to a description of the
 experimental apparatus and the method of observation. The
 results of the experiments are then given, and a discussion
 of the results is presented. The report concludes with a
 summary of the work and a list of references.

The sixth part of the report is devoted to a description of the
 experimental apparatus and the method of observation. The
 results of the experiments are then given, and a discussion
 of the results is presented. The report concludes with a
 summary of the work and a list of references.

The seventh part of the report is devoted to a description of the
 experimental apparatus and the method of observation. The
 results of the experiments are then given, and a discussion
 of the results is presented. The report concludes with a
 summary of the work and a list of references.

The eighth part of the report is devoted to a description of the
 experimental apparatus and the method of observation. The
 results of the experiments are then given, and a discussion
 of the results is presented. The report concludes with a
 summary of the work and a list of references.

The ninth part of the report is devoted to a description of the
 experimental apparatus and the method of observation. The
 results of the experiments are then given, and a discussion
 of the results is presented. The report concludes with a
 summary of the work and a list of references.

The tenth part of the report is devoted to a description of the
 experimental apparatus and the method of observation. The
 results of the experiments are then given, and a discussion
 of the results is presented. The report concludes with a
 summary of the work and a list of references.

NOTA .- El tomar el punto A como origen de división, tiene como consecuencia el conseguir que la cara decagonal superior 1 al 10, la adyacente cuadrada 5-6-16-17 por su parte izquierda, y la adyacente exagonal 1-10-11-22-23-12 por su derecha, queden todas con sus planos perpendiculares a I, con lo cual los diedros correspondientes se obtienen en I en su verdadera magnitud.

Igualmente ocurre en la parte inferior de la figura



e) Radio " r_4 "f) Radio " r_5 "

Los valores analíticos de estos radios los determinaremos posteriormente. Obsérvese que el radio " r_5 " es ligeramente inferior al " a " de la esfera circunscrita, por lo que prácticamente se confunden ambas circunferencias.

6° Obtenida la proyección total en II del arquimedianos, la determinación de la I y III es inmediata. Para ello, tracemos en ambas, paralelas al eje $+X$, equidistantes del centro y con las sucesivas distancias, previamente calculadas, " f_1 ", " f_2 ", " f_3 ", " f_4 " y " f_5 ", sobre las que se encontrarán las proyecciones de todos sus vértices, en correspondencia con las ya obtenidas en III.

Como comprobación y necesaria ayuda para el trazado gráfico dado anteriormente, vamos a determinar analíticamente las siguientes magnitudes complementarias que darán mayor exactitud a dicho trazado.

Apotema " k_6 " de una cara hexagonal

Se demuestra en Geometría, es (ver lám. 42, h. 9)

Para el caso del dibujo,

ent: $k_6 = \frac{\sqrt{3}}{2} \times 14,46 = 12,5 \text{ mm}$

$$k_6 = \frac{\sqrt{3}}{2} l = 0,8660254... l$$

First paragraph of handwritten text, starting with a capital letter, located in the upper section of the page.

Second paragraph of handwritten text, continuing the narrative or list, located in the middle section of the page.

Third paragraph of handwritten text, located in the lower-middle section of the page.

Fourth paragraph of handwritten text, located at the bottom of the page, possibly concluding the entry.

Apotema " k_{10} " de una cara decagonal

Se demuestra en Geometría, es (ver lám. 41, h 12)

$$k_{10} = \frac{\sqrt{5 + 2\sqrt{5}}}{2} l = 1,53 \ 88 \ 41 \ 76 \dots l$$

Para el caso del dibujo, será: $k_{10} = 1,53 \ 88 \ 42 \dots \times 14,46 = 22,3 \text{ mm}$

Distancia " g_1 " de los vértices 11 al 20 al plano de la cara decagonal 1 al 10, y de los vértices 101 al 110 al de la cara decagonal 111 al 120 *

Considerando en I la cara decagonal 1 al 10 y la contigua cuadrada 5-6-17-16 por su parte izquierda, que forman entre sí el ángulo φ_{4-10} , ya conocido, siendo sus respectivos planos perpendiculares a I, se deduce que la altura " g_1 " buscada es la proyección sobre III del eje de la cara cuadrada, siendo el ángulo de proyección:

$$\varphi_{4-10} - 90^\circ = 148^\circ \ 16' \ 57,1'' - 90^\circ = 58^\circ \ 16' \ 57,1'' \text{ de donde}$$

$$g_1 = \cos 58^\circ \ 16' \ 57,1'' \times l = 0,52 \ 57 \ 37 \ 1 \dots l$$

Desarrollo del cálculo anterior:

$$l \cos 58^\circ \ 16' \ 57,1'' = 7,72 \ 07 \ 63 \ 7 \dots = l \cdot 0,52 \ 57 \ 31 \ 1 \dots$$

$$\cos 58^\circ \ 16' \ 57,1'' = 0,52 \ 57 \ 31 \ 1 \dots$$

* Ver cálculo igual en lám. 38, hojas 19 a 22

The first part of the paper is devoted to a discussion of the
 various methods of determining the α and β values of the
 reaction. The α value is determined by the ratio of the
 rate of reaction to the concentration of the reactants. The β value
 is determined by the ratio of the rate of reaction to the
 concentration of the products.

The second part of the paper is devoted to a discussion of the
 various methods of determining the α and β values of the
 reaction. The α value is determined by the ratio of the
 rate of reaction to the concentration of the reactants. The β value
 is determined by the ratio of the rate of reaction to the
 concentration of the products.

The third part of the paper is devoted to a discussion of the
 various methods of determining the α and β values of the
 reaction. The α value is determined by the ratio of the
 rate of reaction to the concentration of the reactants. The β value
 is determined by the ratio of the rate of reaction to the
 concentration of the products.

The fourth part of the paper is devoted to a discussion of the
 various methods of determining the α and β values of the
 reaction. The α value is determined by the ratio of the
 rate of reaction to the concentration of the reactants. The β value
 is determined by the ratio of the rate of reaction to the
 concentration of the products.

$$\frac{d[\text{A}]}{dt} = -k[\text{A}]^{\alpha}[\text{B}]^{\beta}$$

El valor anterior puede ser obtenido más exactamente, mediante el cálculo trigonométrico de los ángulos que intervienen, cuyos valores hemos deducido anteriormente. Su desarrollo es el siguiente:

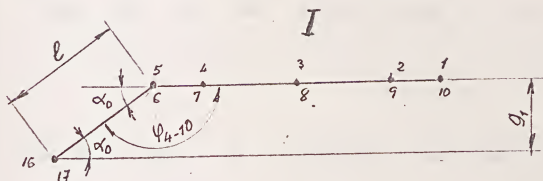


Figura 2

Sea (fig. 2) la proyección parcial en I del Arquimédiano XII, que comprende la cara decagonal 1 al 10 y la contigua ma-

drada (por la parte izquierda) 5-6-17-16, que forman entre sí el ángulo φ_{4-10} , siendo α_0 el ángulo suplementario del mismo. La magnitud de la cota "g" buscada, será pues

$$g_1 = l \operatorname{sen} \alpha_0 \quad [1]$$

pero siendo $\tan \varphi_{4-10} = -\frac{\sqrt{5}-1}{2}$, será $\tan \alpha_0 = \frac{\sqrt{5}-1}{2}$ y por

lo tanto

$$\begin{aligned} \operatorname{sen} \alpha_0 &= \frac{\tan \alpha_0}{\sqrt{1 + \tan^2 \alpha_0}} = \frac{\frac{\sqrt{5}-1}{2}}{\sqrt{1 + \left(\frac{\sqrt{5}-1}{2}\right)^2}} = \frac{\sqrt{5}-1}{2\sqrt{1 + \frac{6-2\sqrt{5}}{4}}} = \frac{\sqrt{5}-1}{2\sqrt{1 + \frac{3-\sqrt{5}}{2}}} = \\ &= \frac{\sqrt{5}-1}{2\sqrt{\frac{5-\sqrt{5}}{2}}} = \frac{1}{2} \sqrt{\frac{(\sqrt{5}-1)^2 \times 2}{5-\sqrt{5}}} = \frac{1}{2} \sqrt{\frac{(6-2\sqrt{5}) \times 2}{5-\sqrt{5}}} = \frac{1}{2} \sqrt{\frac{(3-\sqrt{5}) \times 4}{5-\sqrt{5}}} = \\ &= \sqrt{\frac{3-\sqrt{5}}{5-\sqrt{5}}} = \sqrt{\frac{(3-\sqrt{5})(5+\sqrt{5})}{20}} = \sqrt{\frac{15-5\sqrt{5}+3\sqrt{5}-5}{20}} = \sqrt{\frac{10-2\sqrt{5}}{20}} = \sqrt{\frac{5-\sqrt{5}}{10}} \end{aligned}$$

valor que sustituido en [1], nos da

THE HISTORY OF THE

of the ...

The ...

...

...

...

tanto $\text{sen } \alpha_0 = \frac{t_2 \alpha_0}{\sqrt{1 + t_2^2 \alpha_0}} = \frac{3 - \sqrt{5}}{\sqrt{1 + (3 - \sqrt{5})^2}} = \frac{3 - \sqrt{5}}{\sqrt{1 + (9 + 5 - 6\sqrt{5})}} =$

$$= \frac{3 - \sqrt{5}}{\sqrt{15 - 6\sqrt{5}}} = \sqrt{\frac{(3 - \sqrt{5})^2}{15 - 6\sqrt{5}}} = \sqrt{\frac{14 - 6\sqrt{5}}{15 - 6\sqrt{5}}} = \sqrt{\frac{2(7 - 3\sqrt{5})(15 + 6\sqrt{5})}{225 - 180}} =$$

$$= \sqrt{\frac{2(105 - 45\sqrt{5} + 42\sqrt{5} - 90)}{45}} = \sqrt{\frac{2(15 - 3\sqrt{5})}{45}} = \sqrt{\frac{2(5 - \sqrt{5})}{15}}$$

y siendo $k_6 = \frac{\sqrt{2}}{2} l$ (ver pág. 21), tendremos, substituyendo los
valores en [2]

$$g_1 = k_6 \text{ sen } \alpha_0 = \frac{\sqrt{3}}{2} \times \sqrt{\frac{2(5 - \sqrt{5})}{15}} \times l = \sqrt{\frac{3 \times 2(5 - \sqrt{5})}{4 \times 15}} l =$$

$$= \sqrt{\frac{5 - \sqrt{5}}{10}} l = 0,52573111... l$$

valor coincidente con el obtenido con la cara cuadrada (parte izquierda).

NOTA.- Obsérvese que el valor de " g_2 ", que calcularemos posteriormente seguidos de los de " g_3 " a " g_5 ", se obtiene de inmediato del cálculo anterior, ya que en la figura 2, se deduce que

$$g_2 = 2 k_6 \text{ sen } \alpha_0 = 2 g_1 = 2 \times \sqrt{\frac{5 - \sqrt{5}}{10}} l = \sqrt{\frac{2(5 - \sqrt{5})}{5}} l =$$

$$= \sqrt{\frac{10 - 2\sqrt{5}}{5}} l = 1,0514622... l$$

Los resultados anteriores nos demuestran que los vértices 11 al 20 están en un mismo plano

प्रमाणित किया जाता है कि

श्री _____

राज्य के _____

Distancia " f_1 " entre los dos planos paralelos a II, que contienen los vértices 11 al 20 y 101 al 110, respectivamente.

Se obtiene por diferencia de las alturas " c_{10} " y " g_1 ", ya calculadas.

$$\begin{aligned}
 \boxed{f_1} &= 2 (c_{10} - g_1) = 2 \times \left(\frac{\sqrt{25+10\sqrt{5}}}{2} - \sqrt{\frac{5-\sqrt{5}}{10}} \right) l = \\
 &= 2 \left(\sqrt{\frac{25+10\sqrt{5}}{4}} - \sqrt{\frac{5-\sqrt{5}}{10}} \right) \times l = 2 \sqrt{\frac{25+10\sqrt{5}}{4} + \frac{5-\sqrt{5}}{10} - 2 \times \frac{\sqrt{25+10\sqrt{5}}}{2} \times \sqrt{\frac{5-\sqrt{5}}{10}}} \times l = \\
 &= 2 \sqrt{\frac{125+50\sqrt{5}+10-2\sqrt{5}}{20} - \frac{\sqrt{(5-\sqrt{5})(25+10\sqrt{5})}}{10}} \times l = \\
 &= 2 \sqrt{\frac{135+48\sqrt{5}}{20} - \frac{125-25\sqrt{5}+50\sqrt{5}-50}{10}} \times l = 2 \sqrt{\frac{135+48\sqrt{5}}{20} - \frac{75+25\sqrt{5}}{10}} \times l = \\
 &= 2 \sqrt{\frac{135+48\sqrt{5}}{20} - \frac{5}{\sqrt{10}} \times \sqrt{3+\sqrt{5}}} \times l = 2 \sqrt{\frac{135+48\sqrt{5}}{20} - \frac{5}{\sqrt{10}} \times \left(\sqrt{\frac{5}{2}} + \sqrt{\frac{1}{2}} \right)} \times l = \\
 &= 2 \sqrt{\frac{135+48\sqrt{5}}{20} - 5 \left(\sqrt{\frac{5}{20}} + \sqrt{\frac{1}{20}} \right)} \times l = 2 \sqrt{\frac{135+48\sqrt{5}}{20} - \frac{5}{2} - 5 \times \sqrt{\frac{1}{20}}} \times l = \\
 &= 2 \sqrt{\frac{135+48\sqrt{5}}{20} - \frac{5}{2} - \frac{\sqrt{5}}{2}} \times l = 2 \sqrt{\frac{135+48\sqrt{5}-50-10\sqrt{5}}{20}} \times l = \\
 &= 2 \times \sqrt{\frac{85+38\sqrt{5}}{20}} \times l = \boxed{\sqrt{\frac{85+38\sqrt{5}}{5}}} \times l = 5,83\ 04\ 47\ 38 \dots l
 \end{aligned}$$

Para el caso del dibujo, será: $f_1 = 5,83\ 04\ 47\ 38 \dots \times 14,46 = 84,3\ \text{mm}$

Date	Particulars	Amount
1890	To Balance	100.00
	By Cash	50.00
	By Cash	50.00
	By Cash	50.00
	By Cash	50.00
	By Cash	50.00
	By Cash	50.00
	By Cash	50.00
	By Cash	50.00
	By Cash	50.00
	By Cash	50.00
	By Cash	50.00
	By Cash	50.00
	By Cash	50.00
	By Cash	50.00
	By Cash	50.00
	By Cash	50.00
	By Cash	50.00

Radio "r₁" de las circunferencias que contienen a los vértices
11 al 20 y 101 al 110 respectivamente

Este radio es un cateto de un triángulo rectángulo de hipotenusa "a" y el otro cateto " $\frac{f_1}{2}$ ". Su valor será:

$$\boxed{r_1} = \sqrt{a^2 - \left(\frac{f_1}{2}\right)^2} = \sqrt{\left(\frac{\sqrt{31 + 12\sqrt{5}}}{2} \ell\right)^2 - \left(\frac{\sqrt{85 + 38\sqrt{5}}}{5} \times \frac{1}{2} \ell\right)^2} =$$

$$= \sqrt{\frac{31 + 12\sqrt{5}}{4} - \frac{85 + 38\sqrt{5}}{5 \times 4}} \times \ell = \sqrt{\frac{155 + 60\sqrt{5} - 85 - 38\sqrt{5}}{20}} \ell = \sqrt{\frac{70 + 22\sqrt{5}}{20}} \times \ell =$$

$$\boxed{\sqrt{\frac{35 + 11\sqrt{5}}{10}} \ell} = 2,44124451... \ell$$

Para el caso del dibujo, será: $r_1 = 2,44124451... \times 14,46 = 35,3 \text{ mm}$

Distancia "g₂" de los vértices 21 al 30 al plano de la cara
decaagonal 1 al 10, y de los vértices 91 al 100 a la cara deca-
gonal 111 al 120:

Refiriéndonos a la lámina 44, vemos que la cara deca-
gonal 16-17-22-39-50-51-60-49-38-27, contigua a la cuadra-
da 5-6-17-16, están ambas proyectadas sobre I, según líneas
rectas, por ser sus respectivos planos perpendiculares a I, y
por lo tanto la arista común 16-17, intersección de dichas
caras, será a su vez perpendicular a I.

En la figura 4, representamos el contorno del arqui-

THE UNIVERSITY OF CHICAGO
LIBRARY
1215 EAST 58TH STREET
CHICAGO, ILL. 60637

THE UNIVERSITY OF CHICAGO
LIBRARY

THE UNIVERSITY OF CHICAGO
LIBRARY

THE UNIVERSITY OF CHICAGO
LIBRARY

THE UNIVERSITY OF CHICAGO
LIBRARY

THE UNIVERSITY OF CHICAGO
LIBRARY

THE UNIVERSITY OF CHICAGO
LIBRARY

THE UNIVERSITY OF CHICAGO
LIBRARY

THE UNIVERSITY OF CHICAGO
LIBRARY

THE UNIVERSITY OF CHICAGO
LIBRARY

THE UNIVERSITY OF CHICAGO
LIBRARY

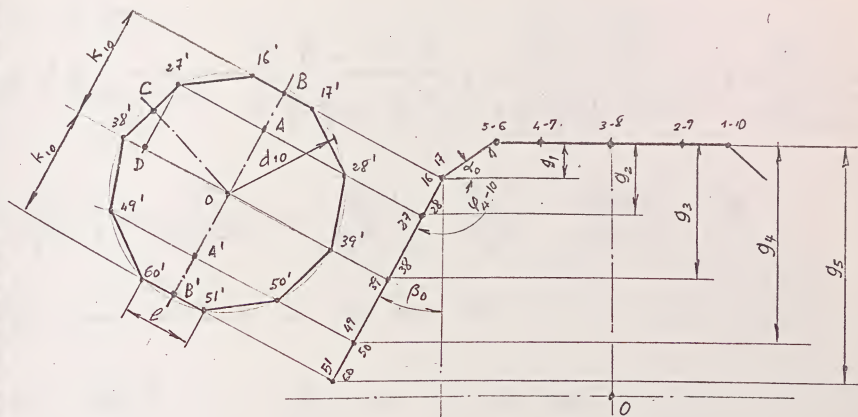


Figura 4

mediano en dicha zona, que incluye la representada en la figura 3. La cara cuadrada 5-6-17-16, tiene contigua la decagonal superior I al 10, paralela a II, y la también decagonal 16-17-22-39-50-51-60-49-38-27, oblicua a II; esta última la hemos representado también abatida sobre el plano del dibujo (parte izquierda de la figura).

De la figura se deduce:

$$\varphi_{4-10} - \alpha_0 = \frac{\pi}{2} + \beta_0 \quad [1]$$

siendo " β_0 " el ángulo de proyección sobre III, de la mencionada cara decagonal oblicua.

De la [1] se deduce:

$$\tan(\varphi_{4-10} - \alpha_0) = \tan\left(\frac{\pi}{2} + \beta_0\right) = -\cot \beta_0 \quad [2]$$

pero ya hemos deducido en el cálculo de " g_1 " que

$$\tan \varphi_{4-10} = -\frac{\sqrt{5}-1}{2} \quad \text{y también} \quad \tan \alpha_0 = \frac{\sqrt{5}-1}{2}$$



The following is a list of the names of the
 persons who have been named in the
 records of the Court, in the year 1800.
 The names are given in the order in which
 they appear in the records, and are not
 necessarily in the order of their birth or
 death.

The names are given in the order in which
 they appear in the records, and are not
 necessarily in the order of their birth or
 death.

The names are given in the order in which
 they appear in the records, and are not
 necessarily in the order of their birth or
 death.

por lo que

$$\begin{aligned} \tan(\varphi_{4-10} - \alpha_0) &= \frac{\tan \varphi_{4-10} - \tan \alpha_0}{1 + \tan \varphi_{4-10} \cdot \tan \alpha_0} = \frac{-\frac{\sqrt{5}-1}{2} - \frac{\sqrt{5}-1}{2}}{1 + \left(-\frac{\sqrt{5}-1}{2}\right)\left(\frac{\sqrt{5}-1}{2}\right)} = \frac{-(\sqrt{5}-1)}{1 - \left(\frac{\sqrt{5}-1}{2}\right)^2} = \\ &= \frac{\sqrt{5}-1}{\left(\frac{\sqrt{5}-1}{2}\right)^2 - 1} = \frac{\sqrt{5}-1}{\frac{5+1-2\sqrt{5}}{4} - 1} = \frac{\sqrt{5}-1}{\frac{3-\sqrt{5}}{2} - 1} = \frac{\sqrt{5}-1}{\frac{1-\sqrt{5}}{2}} = \frac{2(\sqrt{5}-1)}{-(\sqrt{5}-1)} = -2 \end{aligned}$$

Valor que sustituido en [2] nos da

$$-2 = -\cot \beta_0 \quad \text{ " } \quad \cot \beta_0 = 2 \quad \text{ y de aquí:}$$

$$\tan \beta_0 = \frac{1}{2} \quad [3]$$

de esta última se deduce:

$$\boxed{\cos \beta_0} = \frac{1}{\sqrt{1 + \tan^2 \beta_0}} = \frac{1}{\sqrt{1 + \left(\frac{1}{2}\right)^2}} = \frac{1}{\sqrt{1 + \frac{1}{4}}} = \frac{1}{\sqrt{\frac{5}{4}}} = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5} \quad [4]$$

Por otra parte, si en la cara decagonal abatida de la figura 4, trazamos por "O" la perpendicular al lado 38'-27', y por 27' la perpendicular al radio O-38', se nos formarán dos triángulos rectángulos O-C-38' y 27'-D-38' que son semejantes (tienen un ángulo agudo común), por lo que se verificará que

$$\begin{aligned} \frac{\overline{OC}}{O-38'} &= \frac{\overline{27'-D}}{27'-38'} \quad \text{ de donde } \overline{27'-D} = \boxed{AO} = \frac{(27'-38') \times \overline{OC}}{O-38'} = \\ &= \frac{l \times k_{10}}{d_{10}} = \left(\frac{\sqrt{5+2\sqrt{5}}}{2} l : \frac{\sqrt{5}+1}{2} l \right) \times l = \frac{\sqrt{5+2\sqrt{5}}}{\sqrt{5}+1} l = \frac{\sqrt{5+2\sqrt{5}} \times (\sqrt{5}-1)}{4} l = \end{aligned}$$

$\frac{1}{2} \times \frac{3}{4} = \frac{3}{8}$

$\frac{1}{2} \times \frac{3}{4} = \frac{3}{8}$

$\frac{1}{2} \times \frac{3}{4} = \frac{3}{8}$

$\frac{1}{2} \times \frac{3}{4} = \frac{3}{8}$

$\frac{1}{2} \times \frac{3}{4} = \frac{3}{8}$

$\frac{1}{2} \times \frac{3}{4} = \frac{3}{8}$

$\frac{1}{2} \times \frac{3}{4} = \frac{3}{8}$

$$= \frac{\sqrt{(5+2\sqrt{5})(\sqrt{5}-1)^2}}{2} \cdot l = \sqrt{\frac{(5+2\sqrt{5})(6-2\sqrt{5})}{16}} l = \sqrt{\frac{(5+2\sqrt{5})(3-\sqrt{5})}{8}} \cdot l =$$

$$= \sqrt{\frac{15+6\sqrt{5}-5\sqrt{5}-10}{8}} \cdot l = \boxed{\sqrt{\frac{5+\sqrt{5}}{8}} l}$$

De la figura 4, se deduce que

$$\boxed{\overline{BA}} = \overline{BO} - \overline{AO} = k_{10} - \overline{AO} = \frac{\sqrt{5+2\sqrt{5}}}{2} l - \sqrt{\frac{5+\sqrt{5}}{8}} l = \boxed{\sqrt{\frac{5-\sqrt{5}}{8}} l}$$

(ver desarrollo del cálculo anterior en lám. 41, hojas 21 y 22)

De la misma figura 4, se deduce también que

$$\boxed{\overline{BA'}} = \overline{BO} + \overline{AO} = k_{10} + \overline{AO} = \frac{\sqrt{5+2\sqrt{5}}}{2} l + \sqrt{\frac{5+\sqrt{5}}{8}} l = \boxed{\sqrt{\frac{25+11\sqrt{5}}{8}} l}$$

(ver desarrollo del cálculo anterior en lám. 41, hoja 22)

Con los resultados anteriores, podemos obtener los valores siguientes:

$$g_2 = g_1 + \overline{BA} \cos \beta_0 = \sqrt{\frac{5-\sqrt{5}}{10}} l + \sqrt{\frac{5-\sqrt{5}}{8}} \times \frac{2\sqrt{5}}{5} l \quad [5]$$

$$g_3 = g_1 + \overline{BO} \cos \beta_0 = \sqrt{\frac{5-\sqrt{5}}{10}} l + \frac{\sqrt{5+2\sqrt{5}}}{2} \times \frac{2\sqrt{5}}{5} l \quad [6]$$

$$g_4 = g_1 + \overline{BA'} \cos \beta_0 = \sqrt{\frac{5-\sqrt{5}}{10}} l + \sqrt{\frac{25+11\sqrt{5}}{8}} \times \frac{2\sqrt{5}}{5} l \quad [7]$$

$$g_5 = g_1 + \overline{BA'} \cos \beta_0 = \sqrt{\frac{5-\sqrt{5}}{10}} l + 2 \times \frac{\sqrt{5+2\sqrt{5}}}{2} \times \frac{2\sqrt{5}}{5} l \quad [8]$$

El desarrollo de estos cálculos lo haremos progresivamente, comenzando por el de "g₂".

Según la fórmula [5] de la página anterior, tendremos:

$$\begin{aligned}
 [g_2] &= \sqrt{\frac{5-\sqrt{5}}{10}} \cdot l + \sqrt{\frac{5-\sqrt{5}}{8}} \times \frac{2\sqrt{5}}{5} l = \sqrt{\frac{5-\sqrt{5}}{10}} l + \sqrt{\frac{4(5-\sqrt{5})}{8 \times 5}} l = \\
 &= \sqrt{\frac{5-\sqrt{5}}{10}} l + \sqrt{\frac{5-\sqrt{5}}{10}} l = 2 \sqrt{\frac{5-\sqrt{5}}{10}} l = \sqrt{\frac{10-2\sqrt{5}}{5}} l = 1,0514622... l
 \end{aligned}$$

NOTA. - Obsérvese que el valor obtenido para " g_2 ", es coincidente con el ya calculado por diferente camino, en la página n.º 25.

Para el caso del dibujo, será: $g_2 = 1,0514622... \times 14,46 = 15,2 \text{ mm.}$

Distancia " f_2 " entre los dos planos paralelos a II que contienen los vértices 21 al 30 y 91 al 100 respectivamente

Se obtiene por diferencia de las alturas " c_{10} " y " g_2 ", ya calculadas.

$$\begin{aligned}
 [f_2] &= 2(c_{10} - g_2) = 2 \times \left[\frac{\sqrt{35 + 10\sqrt{5}}}{2} l - \sqrt{\frac{10-2\sqrt{5}}{5}} l \right] = \\
 &= \left(\sqrt{35 + 10\sqrt{5}} - 2 \sqrt{\frac{10-2\sqrt{5}}{5}} \right) l = \sqrt{\left(\sqrt{35 + 10\sqrt{5}} - 2 \sqrt{\frac{10-2\sqrt{5}}{5}} \right)^2} \times l = \\
 &= \sqrt{(25 + 10\sqrt{5}) + 4 \times \frac{10-2\sqrt{5}}{5} - 4 \sqrt{\frac{(35+10\sqrt{5})(10-2\sqrt{5})}{5}}} \times l = \\
 &= \sqrt{\frac{125 + 50\sqrt{5} + 40 - 8\sqrt{5}}{5} - 4 \sqrt{\frac{250 + 100\sqrt{5} - 50\sqrt{5} - 100}{5}}} \times l =
 \end{aligned}$$

Handwritten text, likely a letter or document, written in a cursive script. The text is arranged in several paragraphs, with some lines appearing to be numbered or dated. The handwriting is somewhat faded and the ink is light, making it difficult to read accurately. The document appears to be a personal or official communication from the late 19th or early 20th century.

$$\begin{aligned}
 &= \sqrt{\frac{165 + 42\sqrt{5}}{5}} - 4 \sqrt{\frac{150 + 50\sqrt{5}}{5}} \times l = \sqrt{\frac{165 + 42\sqrt{5}}{5}} - 4 \sqrt{30 + 10\sqrt{5}} \times l = \\
 &= \sqrt{\frac{165 + 42\sqrt{5}}{5}} - 4 \sqrt{10} \times \sqrt{3 + \sqrt{5}} \times l = \sqrt{\frac{165 + 42\sqrt{5}}{5}} - 4 \sqrt{10} \left(\sqrt{\frac{5}{2}} + \sqrt{\frac{1}{2}} \right) \times l = \\
 &= \sqrt{\frac{165 + 42\sqrt{5}}{5}} - 4 \sqrt{\frac{50}{2}} - 4 \sqrt{\frac{10}{2}} \times l = \sqrt{\frac{165 + 42\sqrt{5}}{5}} - 20 - 4\sqrt{5} \times l = \\
 &= \sqrt{\frac{165 + 42\sqrt{5} - 100 - 20\sqrt{5}}{5}} \times l = \boxed{\sqrt{\frac{65 + 22\sqrt{5}}{5}}} l = 4,77\ 89\ 85\ 15... l
 \end{aligned}$$

Para el caso del dibujo, será: $f_2 = 4,77\ 89\ 85\ 15... \times 14,46 = 69,1\ \text{mm}$

Radio "r₂" de las circunferencias que contienen a los vértices 21 al 30 y 91 al 100 respectivamente

Este radio es un cateto de un triángulo rectángulo de hipotenusa "a" y el otro cateto "f₂". Su valor será:

$$\begin{aligned}
 r_2 &= \sqrt{a^2 - \left(\frac{f_2}{2}\right)^2} = \sqrt{\left(\frac{\sqrt{31 + 12\sqrt{5}}}{2} l\right)^2 - \left(\frac{\sqrt{65 + 22\sqrt{5}}}{5} \times \frac{1}{2} l\right)^2} = \\
 &= \sqrt{\frac{31 + 12\sqrt{5}}{4} - \frac{65 + 22\sqrt{5}}{20}} \times l = \sqrt{\frac{155 + 60\sqrt{5} - 65 - 22\sqrt{5}}{20}} \times l = \\
 &= \sqrt{\frac{90 + 38\sqrt{5}}{20}} l = \boxed{\sqrt{\frac{45 + 19\sqrt{5}}{10}}} \times l = 2,95\ 77\ 91\ 26... l
 \end{aligned}$$

Para el caso del dibujo, será: $r_2 = 2,95\ 77\ 91\ 26... \times 14,46 = 42,8\ \text{mm}$

1. The first part of the question is to find the value of x and y such that

the matrix $A = \begin{bmatrix} x & y \\ 2 & 3 \end{bmatrix}$ is equal to the matrix $B = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$.

2. The second part of the question is to find the value of x and y such that

the matrix $A = \begin{bmatrix} x & y \\ 2 & 3 \end{bmatrix}$ is equal to the matrix $B = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$.

3. The third part of the question is to find the value of x and y such that

the matrix $A = \begin{bmatrix} x & y \\ 2 & 3 \end{bmatrix}$ is equal to the matrix $B = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$.

4. The fourth part of the question is to find the value of x and y such that

the matrix $A = \begin{bmatrix} x & y \\ 2 & 3 \end{bmatrix}$ is equal to the matrix $B = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$.

5. The fifth part of the question is to find the value of x and y such that

the matrix $A = \begin{bmatrix} x & y \\ 2 & 3 \end{bmatrix}$ is equal to the matrix $B = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$.

6. The sixth part of the question is to find the value of x and y such that

Distancia "g₃" de los vértices 31 al 40 al plano de la cara decaagonal 1 al 10, y de los vértices 81 al 90 a la cara decaagonal 111 al 120.

En la página 30 hemos deducido este valor (ver fórm. [6]), que simplificamos seguidamente.

$$\begin{aligned}
 g_3 &= \sqrt{\frac{5-\sqrt{5}}{10}} l + \frac{\sqrt{5+2\sqrt{5}}}{2} \times \frac{2\sqrt{5}}{5} l = \left(\sqrt{\frac{5-\sqrt{5}}{10}} + \frac{\sqrt{5(5+2\sqrt{5})}}{5} \right) l = \\
 &= \left(\sqrt{\frac{5-\sqrt{5}}{10}} + \sqrt{\frac{5+2\sqrt{5}}{5}} \right) l = \sqrt{\left(\sqrt{\frac{5-\sqrt{5}}{10}} + \sqrt{\frac{5+2\sqrt{5}}{5}} \right)^2} \times l = \\
 &= \sqrt{\frac{5-\sqrt{5}}{10} + \frac{5+2\sqrt{5}}{5} + 2 \sqrt{\frac{(5-\sqrt{5})(5+2\sqrt{5})}{50}}} \times l = \\
 &= \sqrt{\frac{5-\sqrt{5}}{10} + 10 + 4\sqrt{5}} + \frac{2}{5} \sqrt{\frac{25-5\sqrt{5}+10\sqrt{5}-10}{2}} \times l = \sqrt{\frac{15+3\sqrt{5}}{10} + \frac{2}{5} \sqrt{\frac{15+5\sqrt{5}}{2}}} \times l = \\
 &= \sqrt{\frac{15+3\sqrt{5}}{10} + \frac{2\sqrt{5}}{5\sqrt{2}} \times \sqrt{3+\sqrt{5}}} \times l = \sqrt{\frac{15+3\sqrt{5}}{10} + \frac{2\sqrt{5}}{5\sqrt{2}} \times \left(\sqrt{\frac{5}{2}} + \sqrt{\frac{1}{2}} \right)} \times l = \\
 &= \sqrt{\frac{15+3\sqrt{5}}{10} + \frac{2}{5} \sqrt{\frac{25}{4}} + \frac{2}{5} \sqrt{\frac{5}{4}}} \times l = \sqrt{\frac{15+3\sqrt{5}}{10} + \frac{2}{5} \times \frac{5}{2} + \frac{2\sqrt{5}}{10}} \times l = \\
 &= \sqrt{\frac{15+3\sqrt{5}}{10} + \frac{10}{10} + \frac{2\sqrt{5}}{10}} \times l = \sqrt{\frac{15+3\sqrt{5}+10+2\sqrt{5}}{10}} \times l = \sqrt{\frac{25+5\sqrt{5}}{10}} \times l = \\
 &= \sqrt{\frac{5+\sqrt{5}}{2}} \times l = 1,90\ 21\ 13\ 03... l
 \end{aligned}$$

Para el caso del dibujo, será: $g_3 = 1,90\ 21\ 13\ 03... \times 14,46 = 27,5\ \text{mm}$

Let $f(x) = x^2 + 2x + 1$ and $g(x) = x^2 + 1$. Then $f(x) - g(x) = 2x$. Since $2x$ is not a constant, $f(x)$ and $g(x)$ are not linearly independent. However, if we consider $f(x) = x^2 + 2x + 1$ and $g(x) = x^2 + 1$, then $f(x) - g(x) = 2x$, which is a linear function. This suggests that $f(x)$ and $g(x)$ are linearly dependent.

Consider the functions $f(x) = x^2 + 2x + 1$ and $g(x) = x^2 + 1$. We can write $f(x) = g(x) + 2x$. This shows that $f(x)$ is a linear combination of $g(x)$ and x . Therefore, $f(x)$ and $g(x)$ are linearly dependent.

Let $f(x) = x^2 + 2x + 1$ and $g(x) = x^2 + 1$. Then $f(x) - g(x) = 2x$. Since $2x$ is a linear function, $f(x)$ and $g(x)$ are linearly dependent. This is because $f(x)$ can be expressed as a linear combination of $g(x)$ and x .

Consider the functions $f(x) = x^2 + 2x + 1$ and $g(x) = x^2 + 1$. We can write $f(x) = g(x) + 2x$. This shows that $f(x)$ is a linear combination of $g(x)$ and x . Therefore, $f(x)$ and $g(x)$ are linearly dependent.

Let $f(x) = x^2 + 2x + 1$ and $g(x) = x^2 + 1$. Then $f(x) - g(x) = 2x$. Since $2x$ is a linear function, $f(x)$ and $g(x)$ are linearly dependent. This is because $f(x)$ can be expressed as a linear combination of $g(x)$ and x .

Consider the functions $f(x) = x^2 + 2x + 1$ and $g(x) = x^2 + 1$. We can write $f(x) = g(x) + 2x$. This shows that $f(x)$ is a linear combination of $g(x)$ and x . Therefore, $f(x)$ and $g(x)$ are linearly dependent.

Let $f(x) = x^2 + 2x + 1$ and $g(x) = x^2 + 1$. Then $f(x) - g(x) = 2x$. Since $2x$ is a linear function, $f(x)$ and $g(x)$ are linearly dependent. This is because $f(x)$ can be expressed as a linear combination of $g(x)$ and x .

Consider the functions $f(x) = x^2 + 2x + 1$ and $g(x) = x^2 + 1$. We can write $f(x) = g(x) + 2x$. This shows that $f(x)$ is a linear combination of $g(x)$ and x . Therefore, $f(x)$ and $g(x)$ are linearly dependent.

Let $f(x) = x^2 + 2x + 1$ and $g(x) = x^2 + 1$. Then $f(x) - g(x) = 2x$. Since $2x$ is a linear function, $f(x)$ and $g(x)$ are linearly dependent. This is because $f(x)$ can be expressed as a linear combination of $g(x)$ and x .

Consider the functions $f(x) = x^2 + 2x + 1$ and $g(x) = x^2 + 1$. We can write $f(x) = g(x) + 2x$. This shows that $f(x)$ is a linear combination of $g(x)$ and x . Therefore, $f(x)$ and $g(x)$ are linearly dependent.

Distancia " f_3 " entre los dos planos paralelos a II que contienen los vértices 31 al 40 y 81 al 90 respectivamente.

Se obtiene por diferencia de las alturas " c_{10} " y " g_3 " ya calculadas.

$$\begin{aligned}
 \boxed{f_3} &= 2(c_{10} - g_3) = 2 \times \left(\frac{\sqrt{25 + 10\sqrt{5}}}{2} - \sqrt{\frac{5 + \sqrt{5}}{2}} \right) l = \left(\sqrt{25 + 10\sqrt{5}} - \sqrt{2(5 + \sqrt{5})} \right) l = \\
 &= \sqrt{\left(\sqrt{25 + 10\sqrt{5}} - \sqrt{2(5 + \sqrt{5})} \right)^2} \times l = \sqrt{25 + 10\sqrt{5} + 2(5 + \sqrt{5}) - 2\sqrt{(25 + 10\sqrt{5})(5 + \sqrt{5}) \times 2}} l = \\
 &= \sqrt{25 + 10\sqrt{5} + 10 + 2\sqrt{5} - 2\sqrt{(125 + 50\sqrt{5} + 25\sqrt{5} + 50)}} \times l = \\
 &= \sqrt{35 + 12\sqrt{5} - 2\sqrt{2(175 + 75\sqrt{5})}} \times l = \sqrt{35 + 12\sqrt{5} - 2\sqrt{2 \times 25 \times (7 + 3\sqrt{5})}} \times l = \\
 &= \sqrt{35 + 12\sqrt{5} - 10\sqrt{2} \times \sqrt{7 + 3\sqrt{5}}} \times l = \sqrt{35 + 12\sqrt{5} - 10\sqrt{2} \left(\sqrt{\frac{7}{2}} + \sqrt{\frac{3}{2}} \right)} \times l = \\
 &= \sqrt{35 + 12\sqrt{5} - 10 \left(\sqrt{\frac{14}{2}} + \sqrt{\frac{10}{2}} \right)} \times l = \sqrt{35 + 12\sqrt{5} - 10 \times 3 - 10\sqrt{5}} \times l = \boxed{\sqrt{5 + 2\sqrt{5}}} l = \\
 &= 3,07\ 76\ 83\ 53 \dots l
 \end{aligned}$$

Para el caso del dibujo, será: $f_3 = 3,07\ 76\ 83\ 53 \dots \times 14,46 = 44,5\ \text{mm}$

Radio " r_3 " de las circunferencias que contienen a los vértices 31 al 40 y 81 al 90 respectivamente.

Este radio es un cateto de un triángulo rectángulo de hipotenusa " a " y el otro cateto " f_3 ". Su valor será:

$$\begin{aligned}
 \boxed{r_3} &= \sqrt{a^2 - \left(\frac{f_3}{2}\right)^2} = \sqrt{\left(\frac{\sqrt{31+12\sqrt{5}}}{2} \ell\right)^2 - \left(\frac{\sqrt{5+2\sqrt{5}}}{2} \ell\right)^2} = \\
 &= \sqrt{\frac{31+12\sqrt{5}}{4} - \frac{5+2\sqrt{5}}{4}} \ell = \sqrt{\frac{31+12\sqrt{5}-5-2\sqrt{5}}{4}} \ell = \sqrt{\frac{26+10\sqrt{5}}{4}} \ell = \\
 &= \sqrt{\frac{13+5\sqrt{5}}{2}} \ell = 3,47709217... \ell
 \end{aligned}$$

Para el caso del dibujo, será: $3,47709217... \times 14,46 = 50,3 \text{ mm.}$

Distancia "g₄" de los vértices 41 al 50 al plano de la cara decagonal 1 al 10, y de los vértices 71 al 80 a la cara decagonal 111 al 120

En la página 30 hemos deducido este valor (ver fórmula [7]), que simplificaremos seguidamente.

$$\begin{aligned}
 \boxed{g_4} &= \sqrt{\frac{5-\sqrt{5}}{10}} \ell + \sqrt{\frac{25+11\sqrt{5}}{8}} \times \frac{2\sqrt{5}}{5} \ell = \left(\sqrt{\frac{5-\sqrt{5}}{10}} + \frac{2}{5} \sqrt{\frac{125+55\sqrt{5}}{8}} \right) \ell = \\
 &= \left(\sqrt{\frac{5-\sqrt{5}}{10}} + \frac{1}{5} \sqrt{\frac{5(25+11\sqrt{5})}{2}} \right) \ell = \left(\sqrt{\frac{5-\sqrt{5}}{10}} + \sqrt{\frac{25+11\sqrt{5}}{10}} \right) \ell = \\
 &= \sqrt{\left(\sqrt{\frac{5-\sqrt{5}}{10}} + \sqrt{\frac{25+11\sqrt{5}}{10}} \right)^2} \ell = \sqrt{\frac{5-\sqrt{5}}{10} + \frac{25+11\sqrt{5}}{10} + 2 \sqrt{\frac{(5-\sqrt{5})(25+11\sqrt{5})}{10^2}}} \ell = \\
 &= \sqrt{\frac{30+10\sqrt{5}}{10} - \frac{2}{10} \sqrt{125 - 25\sqrt{5} + 55\sqrt{5} - 55}} \ell = \sqrt{3+\sqrt{5} + \frac{1}{5} \sqrt{70+30\sqrt{5}}} \ell = \\
 &= \sqrt{3+\sqrt{5} + \frac{\sqrt{10}}{5} \times \sqrt{7+3\sqrt{5}}} \ell = \sqrt{3+\sqrt{5} + \frac{\sqrt{10}}{5} \left(\sqrt{\frac{7}{2}} + \sqrt{\frac{5}{2}} \right)} \ell = \\
 &= \sqrt{3+\sqrt{5} + \frac{1}{5} \times \sqrt{\frac{70}{2}} + \frac{1}{5} \sqrt{\frac{50}{2}}} \ell = \sqrt{3+\sqrt{5} + \frac{3\sqrt{5}}{5} + 1} \ell =
 \end{aligned}$$

১৯৩৭ সালের ১০/১১/৩৭

১৯৩৭ সালের ১০/১১/৩৭

১৯৩৭ সালের ১০/১১/৩৭

১৯৩৭ সালের ১০/১১/৩৭

১৯৩৭ সালের ১০/১১/৩৭

১৯৩৭ সালের ১০/১১/৩৭

১৯৩৭ সালের ১০/১১/৩৭

১৯৩৭ সালের ১০/১১/৩৭

১৯৩৭ সালের ১০/১১/৩৭

১৯৩৭ সালের ১০/১১/৩৭

১৯৩৭ সালের ১০/১১/৩৭

১৯৩৭ সালের ১০/১১/৩৭

$$= \sqrt{\frac{15 + 5\sqrt{5} + 3\sqrt{5} + 5}{5}} l = \sqrt{\frac{20 + 8\sqrt{5}}{5}} \times l = \boxed{2 \sqrt{\frac{5 + 2\sqrt{5}}{5}}} l =$$

$$= 2, 75 \ 27 \ 63 \ 84 \dots l$$

Para el caso del dibujo, será: $g_4 = 2, 75 \ 27 \ 63 \ 84 \dots \times 14,46 = 39,8 \text{ m}$

Distancia "f₄" entre los dos planos paralelos a II que contienen los vértices 41 al 50 y 71 al 80 respectivamente.

Se obtiene por diferencia de las alturas "C₁₀" y "g₄", ya calculadas.

$$\boxed{f_4} = 2 (C_{10} - g_4) = 2 \left(\frac{\sqrt{25 + 10\sqrt{5}}}{2} - 2 \sqrt{\frac{5 + 2\sqrt{5}}{5}} \right) l =$$

$$= \left(\sqrt{25 + 10\sqrt{5}} - 4 \sqrt{\frac{5 + 2\sqrt{5}}{5}} \right) l = \left(\sqrt{5(5 + 2\sqrt{5})} - 4 \sqrt{\frac{5 + 2\sqrt{5}}{5}} \right) l =$$

$$= \left(\sqrt{\frac{25(5 + 2\sqrt{5})}{5}} - 4 \sqrt{\frac{5 + 2\sqrt{5}}{5}} \right) l = \left(5 \sqrt{\frac{5 + 2\sqrt{5}}{5}} - 4 \sqrt{\frac{5 + 2\sqrt{5}}{5}} \right) l = \boxed{\sqrt{\frac{5 + 2\sqrt{5}}{5}}} l =$$

$$= 1, 37 \ 63 \ 81 \ 92 \dots l$$

Para el caso del dibujo, será: $f_4 = 1, 37 \ 63 \ 81 \ 92 \dots \times 14,46 =$

Radio "r₄" de las circunferencias que contienen a los vértices 41 al 50 y 71 al 80 respectivamente

Este radio es un cateto de un triángulo rectángulo de hipotenusa "a" y el otro cateto "f₄". Su valor

1. $\frac{1}{x^2} = x^{-2}$

2. $\frac{d}{dx} x^{-2} = -2x^{-3}$

3. $= -2x^{-3} = -\frac{2}{x^3}$

4. $\frac{d}{dx} \frac{1}{x^2} = -\frac{2}{x^3}$

5. $\frac{d}{dx} \frac{1}{x^2} = -\frac{2}{x^3}$

6. $\frac{d}{dx} \frac{1}{x^2} = -\frac{2}{x^3}$

7. $\frac{d}{dx} \frac{1}{x^2} = -\frac{2}{x^3}$

8. $\frac{d}{dx} \frac{1}{x^2} = -\frac{2}{x^3}$

9. $\frac{d}{dx} \frac{1}{x^2} = -\frac{2}{x^3}$

10. $\frac{d}{dx} \frac{1}{x^2} = -\frac{2}{x^3}$

11. $\frac{d}{dx} \frac{1}{x^2} = -\frac{2}{x^3}$

$$\begin{aligned}
 \text{area: } \boxed{r_4} &= \sqrt{a^2 - \left(\frac{f_4}{2}\right)^2} = \sqrt{\left(\frac{\sqrt{31+12\sqrt{5}}}{2} \ell\right)^2 - \left(\frac{1}{2} \sqrt{\frac{5+2\sqrt{5}}{5}} \ell\right)^2} = \\
 &= \sqrt{\frac{31+12\sqrt{5}}{4} - \frac{1}{4} \times \frac{5+2\sqrt{5}}{5}} \times \ell = \sqrt{\frac{155 + 60\sqrt{5} - 5 - 2\sqrt{5}}{20}} \times \ell = \sqrt{\frac{150 + 58\sqrt{5}}{20}} \ell = \\
 &= \sqrt{\frac{75 + 29\sqrt{5}}{10}} \ell = 3,73 \ 95 \ 98 \ 53... \ell
 \end{aligned}$$

Para el caso del dibujo, area: $r_4 = 3,73 \ 95 \ 98 \ 53... \times 14,46 = 54,1 \text{ mm}$

Distancia "g₅" de los vértices 51 al 60 al plano de la cara
decagonal 1 al 10, y de los vértices 61 al 70 a la cara dec-
gonal 111 al 120.

En la página 30 hemos deducido este valor (ver fórmula [8]),
que simplificamos seguidamente.

$$\begin{aligned}
 \boxed{g_5} &= \sqrt{\frac{5-\sqrt{5}}{10}} \ell + 2 \times \frac{\sqrt{5+2\sqrt{5}}}{2} \times \frac{2\sqrt{5}}{5} \ell = \left(\sqrt{\frac{5-\sqrt{5}}{10}} + \sqrt{\frac{(5+2\sqrt{5}) \times 2^2 \times 5}{5^2}} \right) \ell = \\
 &= \left(\sqrt{\frac{5-\sqrt{5}}{10}} + \sqrt{\frac{4(5+2\sqrt{5})}{5}} \right) \ell = \sqrt{\left(\sqrt{\frac{5-\sqrt{5}}{10}} + \sqrt{\frac{4(5+2\sqrt{5})}{5}} \right)^2} \times \ell = \\
 &= \sqrt{\frac{5-\sqrt{5}}{10} + \frac{4(5+2\sqrt{5})}{5} + 2 \sqrt{\frac{4(5+2\sqrt{5})(5-\sqrt{5})}{50}}} \ell = \\
 &= \sqrt{\frac{5-\sqrt{5}}{10} + \frac{40+16\sqrt{5}}{5} + \frac{2}{5} \sqrt{2(25+10\sqrt{5}-5\sqrt{5}-10)}} \times \ell = \\
 &= \sqrt{\frac{45+15\sqrt{5}}{10} + \frac{2}{5} \sqrt{2(15+5\sqrt{5})}} \times \ell = \sqrt{\frac{9+3\sqrt{5}}{2} + \frac{2}{5} \sqrt{10(3+\sqrt{5})}} \times \ell = \\
 &= \sqrt{\frac{9+3\sqrt{5}}{2} + \frac{2}{5} \sqrt{10} \times \left(\sqrt{\frac{5}{2}} + \sqrt{\frac{1}{2}} \right)} \times \ell = \sqrt{\frac{9+3\sqrt{5}}{2} + \frac{2}{5} \sqrt{\frac{50}{2}} + \frac{2}{5} \sqrt{\frac{10}{2}}} \times \ell =
 \end{aligned}$$

சென்னை நகராட்சி, 1947

பெயர்: சென்னை நகராட்சி, இலக்கு: 1947

தமிழ் மொழி

சென்னை நகராட்சி, 1947

சென்னை நகராட்சி, 1947

சென்னை நகராட்சி, 1947

சென்னை நகராட்சி, 1947

சென்னை நகராட்சி, 1947

சென்னை நகராட்சி, 1947

சென்னை நகராட்சி, 1947

சென்னை நகராட்சி, 1947

சென்னை நகராட்சி, 1947

$$= \sqrt{\frac{9 + 3\sqrt{5}}{2}} + 2 + \frac{2\sqrt{5}}{5} \times l = \sqrt{\frac{45 + 15\sqrt{5} + 20 + 4\sqrt{5}}{10}} \times l = \boxed{\sqrt{\frac{65 + 19\sqrt{5}}{10}} \times l} =$$

$$= 3,37849495... l$$

Para el caso del dibujo, sea: $g_5 = 3,37849495... \times 14,46 = 47,4 \text{ mm}$

Distancia "f₅" entre los dos planos paralelos a II que contienen los vértices 51 al 60 y 61 al 70 respectivamente.

Se obtiene por diferencia de las alturas "C₁₀" y "g₅", ya calculados.

$$\begin{aligned} \boxed{f_5} &= 2(C_{10} - g_5) = 2\left(\frac{\sqrt{25 + 10\sqrt{5}}}{2} - \sqrt{\frac{65 + 19\sqrt{5}}{10}}\right) l = \\ &= (\sqrt{25 + 10\sqrt{5}} - 2\sqrt{\frac{65 + 19\sqrt{5}}{10}}) l = \sqrt{(\sqrt{25 + 10\sqrt{5}} - 2\sqrt{\frac{65 + 19\sqrt{5}}{10}})^2} \times l = \\ &= \sqrt{25 + 10\sqrt{5} + 4 \times \frac{65 + 19\sqrt{5}}{10} - 4\sqrt{\frac{(25 + 10\sqrt{5})(65 + 19\sqrt{5})}{10}}} \times l \\ &= \sqrt{25 + 10\sqrt{5} + \frac{130 + 38\sqrt{5}}{5} - 4\sqrt{\frac{1625 + 650\sqrt{5} + 475\sqrt{5} + 950}{10}}} \times l = \\ &= \sqrt{\frac{125 + 50\sqrt{5} + 130 + 38\sqrt{5}}{5} - 4\sqrt{\frac{2575 + 1125\sqrt{5}}{10}}} \times l = \sqrt{\frac{255 + 88\sqrt{5}}{5} - 4\sqrt{\frac{515 + 225}{2}}} \times l = \\ &= \sqrt{\frac{255 + 88\sqrt{5}}{5} - 4\sqrt{\frac{5(103 + 45\sqrt{5})}{2}}} \times l = \sqrt{\frac{255 + 88\sqrt{5}}{5} - 4\sqrt{\frac{5}{2}} \times \sqrt{103 + 45\sqrt{5}}} \times l \\ &= \sqrt{\frac{255 + 88\sqrt{5}}{5} - 4\sqrt{\frac{5}{2}} \left(\sqrt{\frac{103 + 22}{2}} + \sqrt{\frac{103 - 22}{2}}\right)} \times l = \\ &= \sqrt{\frac{255 + 88\sqrt{5}}{5} - 4\sqrt{\frac{5}{2}} \times \sqrt{\frac{125}{2}} - 4\sqrt{\frac{5}{2}} \times \sqrt{\frac{81}{2}}} \times l = \end{aligned}$$

1. Introduction

The purpose of this study is to investigate the effects of the independent variable on the dependent variable.

The study was conducted in a laboratory setting with a sample size of 30 participants.

The results of the study indicate that there is a significant positive correlation between the independent variable and the dependent variable.

The findings suggest that the independent variable has a positive impact on the dependent variable.

The study was limited by the sample size and the laboratory setting.

Future research should investigate the effects of the independent variable on the dependent variable in a natural setting.

The study was approved by the Institutional Review Board.

The data were analyzed using statistical software.

The results are presented in the following tables and figures.

The study was funded by the National Science Foundation.

The author would like to thank the participants for their contribution to the study.

The author would like to thank the reviewers for their comments.

$$= \sqrt{\frac{255 + 88\sqrt{5}}{5}} - 4 \sqrt{\frac{125 \times 5}{2 \times 2}} - 4 \sqrt{\frac{81 \times 5}{3 \times 2}} \times l = \sqrt{\frac{255 + 88\sqrt{5}}{2}} - 50 - 18\sqrt{5} \times l =$$

$$= \sqrt{\frac{255 + 88\sqrt{5} - 250 - 70\sqrt{5}}{5}} \cdot l = \boxed{\sqrt{\frac{5 - 2\sqrt{5}}{5}} l} = 0,32491970... l$$

Para el caso del dibujo, será: $f_5 = 0,3249197... \times 14,46 = 4,7 \text{ mm}$

Distancia "r₅" de las circunferencias que contienen a los vértices 51 al 60 y 61 al 70 respectivamente.

Este radio es un cateto de un triángulo rectángulo de hipotenusa "a" y el otro cateto "f₅". Su valor será:

$$\boxed{r_5} = \sqrt{a^2 - \left(\frac{f_5}{2}\right)^2} = \sqrt{\left(\frac{31 + 12\sqrt{5}}{2} l\right)^2 - \left(\frac{1}{2} \sqrt{\frac{5 - 2\sqrt{5}}{5}} l\right)^2} =$$

$$= \sqrt{\frac{31 + 12\sqrt{5}}{4} - \frac{1}{4} \times \frac{5 - 2\sqrt{5}}{5}} \cdot l = \sqrt{\frac{155 + 60\sqrt{5} - 5 + 2\sqrt{5}}{20}} \cdot l = \sqrt{\frac{150 + 62\sqrt{5}}{20}} l =$$

$$= \boxed{\sqrt{\frac{75 + 31\sqrt{5}}{10}} \cdot l} = 3,79892231... l$$

Para el caso del dibujo, será: $r_5 = 3,79892231... \times 14,46 = 54,9 \text{ mm}$

(prácticamente este radio es igual al de la esfera circunscrita al arquimedianos)

En el cuadro sinóptico que damos a continuación, resumimos los resultados de los valores complementarios deducidos.

1. The first part of the paper is devoted to the study of the properties of the function $f(x)$ defined by the equation

$$f(x) = \frac{1}{x} \int_0^x f(t) dt$$

where $f(x)$ is a continuous function on the interval $[0, 1]$ and $f(0) = 1$.

It is easy to see that the function $f(x)$ is continuous on the interval $[0, 1]$ and that $f(0) = 1$.

Let us now consider the function $f(x)$ on the interval $(0, 1]$. We shall show that $f(x)$ is a decreasing function on this interval.

$$f'(x) = -\frac{1}{x^2} \int_0^x f(t) dt + \frac{1}{x} f(x)$$

Since $f(x) > 0$ on the interval $(0, 1]$, we have $f'(x) < 0$ on this interval.

$$f(x) = \frac{1}{x} \int_0^x f(t) dt$$

Let us now consider the function $f(x)$ on the interval $(0, 1]$. We shall show that $f(x)$ is a decreasing function on this interval.

Let us now consider the function $f(x)$ on the interval $(0, 1]$. We shall show that $f(x)$ is a decreasing function on this interval.

Let us now consider the function $f(x)$ on the interval $(0, 1]$. We shall show that $f(x)$ is a decreasing function on this interval.

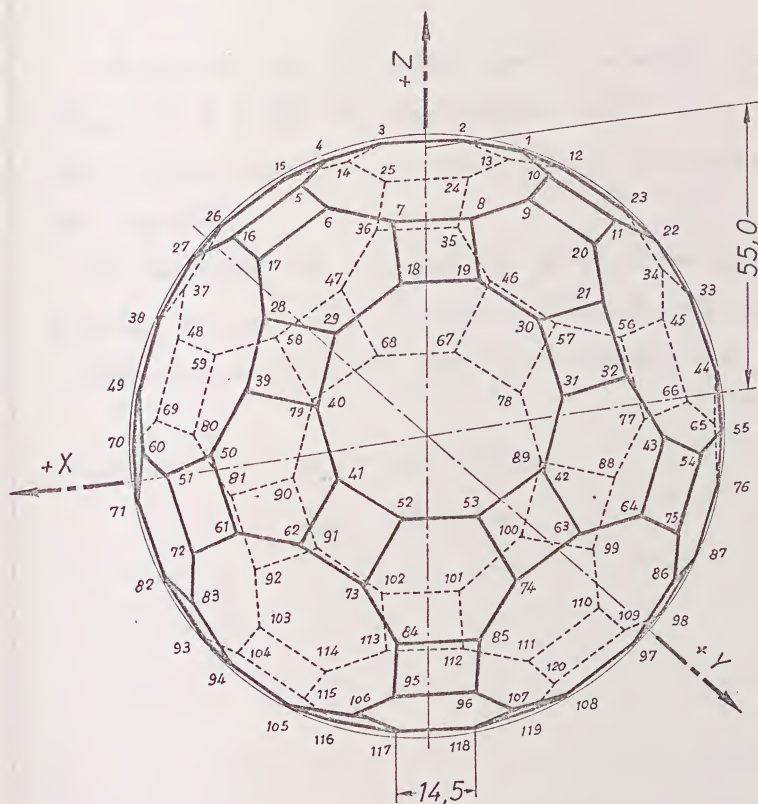
CUADRO SINÓPTICO DE LAS MAGNITUDES COMPLEMENTARIAS

Magnitud	Valor exacto	Valor decimal aproximado
k_6	$\frac{\sqrt{3}}{2} \ell$	0, 86 60 25... ℓ
k_{10}	$\frac{\sqrt{5+2\sqrt{5}}}{2} \ell$	1, 53 88 42... ℓ
f_1	$\sqrt{\frac{85+38\sqrt{5}}{5}} \ell$	5, 83 04 47... ℓ
f_2	$\sqrt{\frac{65+22\sqrt{5}}{5}} \ell$	4, 77 89 85... ℓ
f_3	$\sqrt{5+2\sqrt{5}} \ell$	3, 07 76 84... ℓ
f_4	$\sqrt{\frac{5+2\sqrt{5}}{5}} \ell$	1, 37 63 82... ℓ
f_5	$\sqrt{\frac{5-2\sqrt{5}}{5}} \ell$	0, 32 49 20... ℓ
g_1	$\sqrt{\frac{5-\sqrt{5}}{10}} \ell$	0, 52 57 31... ℓ
g_2	$\sqrt{\frac{10-2\sqrt{5}}{5}} \ell$	1, 05 14 62... ℓ
g_3	$\sqrt{\frac{5+\sqrt{5}}{2}} \ell$	1, 90 21 13... ℓ
g_4	$2\sqrt{\frac{5+2\sqrt{5}}{5}} \ell$	2, 75 27 64... ℓ
g_5	$\sqrt{\frac{65+19\sqrt{5}}{10}} \ell$	3, 27 84 95... ℓ
Γ_1	$\sqrt{\frac{35+11\sqrt{5}}{10}} \ell$	2, 44 12 45... ℓ
Γ_2	$\sqrt{\frac{45+19\sqrt{5}}{10}} \ell$	2, 95 77 91... ℓ
Γ_3	$\sqrt{\frac{13+5\sqrt{5}}{2}} \ell$	3, 47 70 92... ℓ
Γ_4	$\sqrt{\frac{75+29\sqrt{5}}{10}} \ell$	3, 73 95 99... ℓ
Γ_5	$\sqrt{\frac{75+31\sqrt{5}}{10}} \ell$	3, 79 89 22... ℓ



Table 1. Summary of the results of the experiments.

Experiment	Time (min)	Result
1. 100 mg/kg	10	100%
2. 200 mg/kg	20	200%
3. 300 mg/kg	30	300%
4. 400 mg/kg	40	400%
5. 500 mg/kg	50	500%
6. 600 mg/kg	60	600%
7. 700 mg/kg	70	700%
8. 800 mg/kg	80	800%
9. 900 mg/kg	90	900%
10. 1000 mg/kg	100	1000%
11. 1100 mg/kg	110	1100%
12. 1200 mg/kg	120	1200%
13. 1300 mg/kg	130	1300%
14. 1400 mg/kg	140	1400%
15. 1500 mg/kg	150	1500%
16. 1600 mg/kg	160	1600%
17. 1700 mg/kg	170	1700%
18. 1800 mg/kg	180	1800%
19. 1900 mg/kg	190	1900%
20. 2000 mg/kg	200	2000%



Arquimediano XII



ENUNCIADO

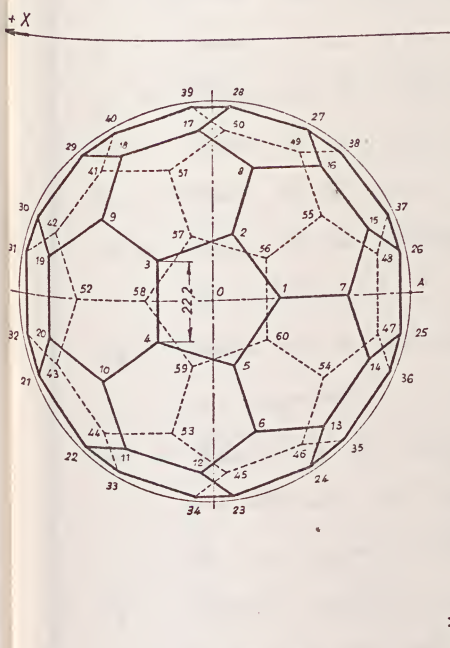
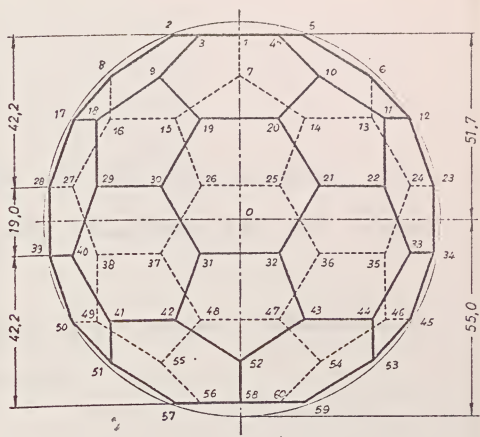
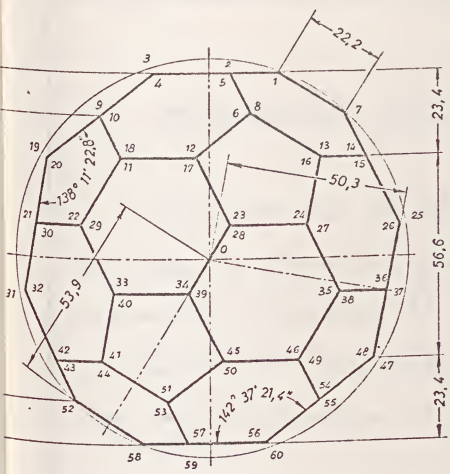
Representar por el método gráfico-analítico, en los planos I, II y III, el Arquimediano XIII, en el que en cada vértice concurren un pentágono y dos octágonos, todos regulares.

La longitud de su lado es de 22,2 mm, y las coordenadas de su centro O, son: O (72, 72, 85) mm.

Dibujar en formato A3V y a escala 1:1.

DATOS: O (72, 72, 85) mm

$l_{XIII} = 22,2 \text{ mm}$



ARQUIMEDIANO XIII

- Número de caras pentagonales..... $C_5 = 12$
- Número de caras exagonales..... $C_6 = 20$
- Número de vértices..... $V = 60$
- Número de aristas..... $A = 90$
- Número de caras de un ángulo sólido..... $1P_5 + 2P_6$

ENUNCIADO

Representar por el método gráfico-analítico, en los planos I, II y III el... Arquimediano XIII, en el que en cada vértice concurren un pentágono y dos exágonos regulares.

La longitud de su lado es de 22,2... mm y las coordenadas de su centro.. O, son O(72, 72, 85) mm.

Dibujar en formato A3v y a escala 1:1.

Propuesta	De entrega	Entregada	Calificación	(firma)	Escuela
Fecha:					Curso
Alumno:					
Escala	Arquimediano XIII				Lámina 45
1:1					Curso 19 - 19



1. The first part of the paper is devoted to a general discussion of the problem. It is shown that the problem is of great importance in the theory of the structure of matter. The author then proceeds to a detailed analysis of the problem, showing that it is of great importance in the theory of the structure of matter.

The author then proceeds to a detailed analysis of the problem, showing that it is of great importance in the theory of the structure of matter. The author then proceeds to a detailed analysis of the problem, showing that it is of great importance in the theory of the structure of matter.



CONSIDERACIONES PREVIAS

Seguiremos en el estudio de este arquimedianos, las directrices y fórmulas generales planteadas en el "Arquimedianos I", lámina 33.

En el caso particular que nos ocupa, determinaremos las magnitudes siguientes:

l = Arista del Arquimedianos XIII (dato del ejercicio)

a = Radio de la esfera circunscrita.

b = Radio de la esfera tangente a las aristas.

c_5 = Radio de la esfera tangente a las caras pentagonales

c_6 = Radio de la esfera tangente a las caras hexagonales

d_5 = Radio de la circunferencia circunscrita a una cara pentagonal

d_6 = Radio de la circunferencia circunscrita a una cara hexagonal.

m = Radio de la circunferencia circunscrita al polígono obtenido al unir los extremos de las aristas de un ángulo sólido.

α_5 = Ángulo rectilíneo del diedro formado por una cara pentagonal, con el plano diametral del arquimedianos que pasa por una arista de aquella.

α_6 = Ángulo rectilíneo del diedro formado por una

cara exagonal, con el plano diametral del arquimedianos que pasa por una arista de aquélla.

φ_{5-6} = Ángulo rectilíneo del diedro formado por una cara pentagonal y otra exagonal.

φ_{6-6} = Ángulo rectilíneo del diedro formado por dos caras exagonales.

S = Superficie

V = Volumen.

PROCESO GRÁFICO - ANALÍTICO

El estudio realizado de este arquimedianos, nos indica que se compone de 12 caras pentagonales y 20 caras exagonales; 60 vértices y 90 aristas.

En cada vértice concurren un pentágono y dos exágonos, todos regulares y de igual lado "l".

Así pues, tendremos que:

$\text{ARQUIMEDIANO XIII } (1P_5 + 2P_6); C_5 = 12; C_6 = 20; V = 60; A = 90$

Cálculo de sus magnitudes

Arista "l" del arquimedianos

Dato del ejercicio

Radio "m" de la circunferencia circunscrita al polígono obtenido al unir los extremos de las tres aristas que concurren en un ángulo sólido.

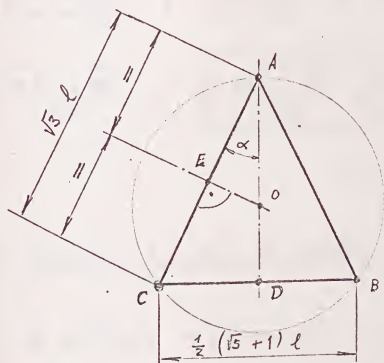


Figura 1

Dicho polígono (fig. 1) es un triángulo isósceles, cuya base BC es la diagonal de una cara pentagonal, y sus otros dos lados iguales $AC = AB$, corresponden a la diagonal de una cara hexagonal.

Se demuestra en Geometría que la diagonal de un pentágono regular, en función de su lado "l", es

$$\overline{CB} = \frac{\sqrt{5} + 1}{2} l \quad \text{y por lo tanto} \quad \overline{CD} = \frac{\sqrt{5} + 1}{4} l$$

y la de un hexágono regular, también de lado "l", es

$$\overline{AC} = \overline{AB} = \sqrt{3} l$$

De la figura se deduce:

$$\begin{aligned} \overline{AD} &= \sqrt{\overline{AC}^2 - \overline{CD}^2} = \sqrt{(\sqrt{3} l)^2 - \left(\frac{\sqrt{5} + 1}{4} l\right)^2} = \sqrt{3 - \left(\frac{\sqrt{5} + 1}{4}\right)^2} \cdot l = \\ &= \sqrt{3 - \frac{6 + 2\sqrt{5}}{16}} \cdot l = \sqrt{\frac{48 - 6 - 2\sqrt{5}}{16}} l = \sqrt{\frac{42 - 2\sqrt{5}}{16}} l = \sqrt{\frac{21 - \sqrt{5}}{8}} l \end{aligned}$$

$$\text{por lo que será:} \quad \cos \alpha = \frac{\overline{AD}}{\overline{AC}} = \frac{\sqrt{\frac{21 - \sqrt{5}}{8}} l}{\sqrt{3} l} = \sqrt{\frac{21 - \sqrt{5}}{24}}$$

Q.1. A triangle ABC is shown in the figure. The side BC is extended to D such that BC = CD. A line segment AD is drawn. Prove that $\angle A < \angle ACD$.



Ans. In $\triangle ABC$ and $\triangle ACD$,
 $BC = CD$ (Given)
 $AC = AC$ (Common side)
 $\angle ACB = \angle ACD$ (Vertically opposite angles)
 $\therefore \triangle ABC \cong \triangle ACD$ (SAS)
 $\therefore \angle B = \angle D$ (Corresponding angles)
 $\therefore \angle A < \angle ACD$ (Exterior angle is greater than the interior opposite angle).

Q.2. In the figure, $AB \parallel CD$ and $AD \parallel BC$. Prove that $\angle A = \angle C$.

Ans. In $\triangle ABC$ and $\triangle CDA$,
 $\angle B = \angle D$ (Opposite angles of a parallelogram)
 $AB = CD$ (Opposite sides of a parallelogram)
 $BC = DA$ (Opposite sides of a parallelogram)
 $\therefore \triangle ABC \cong \triangle CDA$ (SSS)
 $\therefore \angle A = \angle C$ (Corresponding angles).

Q.3. In the figure, $AB \parallel CD$ and $AD \parallel BC$. Prove that $\angle A + \angle C = 180^\circ$.

Ans. In $\triangle ABC$ and $\triangle CDA$,
 $\angle B = \angle D$ (Opposite angles of a parallelogram)
 $AB = CD$ (Opposite sides of a parallelogram)
 $BC = DA$ (Opposite sides of a parallelogram)
 $\therefore \triangle ABC \cong \triangle CDA$ (SSS)
 $\therefore \angle A = \angle C$ (Corresponding angles)

Q.4. In the figure, $AB \parallel CD$ and $AD \parallel BC$. Prove that $\angle A + \angle B = 180^\circ$.

y en consecuencia:

$$\begin{aligned}\overline{40} &= \boxed{m} = \frac{\overline{4E}}{\cos \alpha} = \frac{\sqrt{3}}{2} l : \sqrt{\frac{21-\sqrt{5}}{24}} = \sqrt{\frac{3}{4} : \frac{21-\sqrt{5}}{24}} l = \\ &= \sqrt{\frac{3 \times 24}{4(21-\sqrt{5})}} l = \sqrt{\frac{3 \times 6}{21-\sqrt{5}}} l = \sqrt{\frac{18(21+\sqrt{5})}{21^2-5}} l = \sqrt{\frac{18(21+\sqrt{5})}{436}} l \\ &= \sqrt{\frac{9(21+\sqrt{5})}{218}} l = \boxed{3 \cdot \sqrt{\frac{21+\sqrt{5}}{218}}} l = 0,97943208... l\end{aligned}$$

Radio "a" de la esfera circunscrita

Se obtiene aplicando la fórmula general [1] (ver lám. 33)

$$\begin{aligned}\boxed{a} &= \frac{l^2}{2\sqrt{l^2-m^2}} = \frac{1}{2\sqrt{1-\frac{9(21+\sqrt{5})}{218}}} \times l = \frac{1}{2\sqrt{\frac{218-189-9\sqrt{5}}{218}}} \times l = \\ &= \frac{1}{2\sqrt{\frac{29-9\sqrt{5}}{218}}} \times l = \frac{1}{\sqrt{\frac{4(29-9\sqrt{5})}{218}}} l = \frac{1}{\sqrt{\frac{2(29-9\sqrt{5})}{109}}} \times l = \sqrt{\frac{109}{2(29-9\sqrt{5})}} l = \\ &= \sqrt{\frac{109(29+9\sqrt{5})}{2 \times (29^2-9^2 \times 5)}} \times l = \sqrt{\frac{109(29+9\sqrt{5})}{2 \times 436}} \times l = \sqrt{\frac{29+9\sqrt{5}}{2 \times 4}} \times l = \boxed{\sqrt{\frac{29+9\sqrt{5}}{8}}} l = \\ &= 2,47801866... l\end{aligned}$$

Para el caso del dibujo, sea: $a = 55 \text{ mm}$ $l = 22,195 \text{ mm}$.

Radio "b" de la esfera tangente a las aristas

Handwritten text in Devanagari script, consisting of several lines of prose.

Handwritten text in Devanagari script, continuing the narrative or list.

Handwritten text in Devanagari script, appearing as a separate section or entry.

Handwritten text in Devanagari script, continuing the content.

Handwritten text in Devanagari script, possibly a concluding statement or signature.

Handwritten text in Devanagari script, appearing as a final line or note.

Se obtiene aplicando la fórmula general [3] (ver lám. 33)

$$\begin{aligned}
 \boxed{b} &= \sqrt{a^2 - \frac{l^2}{4}} = \sqrt{\left(\sqrt{\frac{29+9\sqrt{5}}{8}} l\right)^2 - \frac{l^2}{4}} = \sqrt{\frac{29+9\sqrt{5}}{8} - \frac{1}{4}} \cdot l = \\
 &= \sqrt{\frac{29+9\sqrt{5}-2}{8}} \cdot l = \sqrt{\frac{27+9\sqrt{5}}{8}} \cdot l = \sqrt{\frac{9(3+\sqrt{5})}{8}} \cdot l = \frac{3}{2} \sqrt{\frac{3+\sqrt{5}}{2}} \cdot l = \\
 &= \frac{3}{2} \times \frac{\sqrt{3+\sqrt{5}}}{\sqrt{2}} \cdot l = \frac{3}{2} \times \frac{\sqrt{\frac{5}{2}} + \sqrt{\frac{1}{2}}}{\sqrt{2}} \cdot l = \frac{3}{2} \left(\sqrt{\frac{5}{4}} + \sqrt{\frac{1}{4}} \right) \cdot l = \frac{3}{2} \times \left(\frac{\sqrt{5}}{2} + \frac{1}{2} \right) \cdot l = \\
 &= \left(\frac{3\sqrt{5}}{4} + \frac{3}{4} \right) \cdot l = \boxed{\frac{3(\sqrt{5}+1)}{4}} \cdot l = 2,42705098... \cdot l
 \end{aligned}$$

Para el caso del dibujo, será: $b = 2,42705098... \times 22,195 = 53,9 \text{ mm}$

Radio "d₅" de la circunferencia circunscrita a una
una pentagonal. de lado "l"

Se demuestra en Geometría, es

$$\boxed{d_5 = \sqrt{\frac{5+\sqrt{5}}{10}} \cdot l} = 0,8506508... \cdot l$$

Para el caso del dibujo, será: $d_5 = 0,8506508... \times 22,195 = 18,9 \text{ mm}$

Radio "d₆" de la circunferencia circunscrita a una ca-
ca exagonal de lado "l"

Se demuestra en Geometría, es

$$\boxed{d_6 = l}$$

Let x and y be real numbers such that $x^2 + y^2 = 1$.

$$(x^2 + y^2)^2 = 1^2 = 1$$

$$(x^2 + y^2)^2 = x^4 + 2x^2y^2 + y^4 = 1$$

$$x^4 + 2x^2y^2 + y^4 = 1$$

$$x^4 + y^4 + 2x^2y^2 = 1$$

Since $x^2 + y^2 = 1$, we have $x^2 = 1 - y^2$ and $y^2 = 1 - x^2$.

$$x^2 = 1 - y^2$$

Substituting $x^2 = 1 - y^2$ into $x^4 + y^4 + 2x^2y^2 = 1$, we get

$$(1 - y^2)^2 + y^4 + 2(1 - y^2)y^2 = 1$$

$$1 - 2y^2 + y^4 + y^4 + 2y^2 - 2y^4 = 1$$

$$1 - 2y^2 + y^4 + y^4 + 2y^2 - 2y^4 = 1$$

Simplifying, we have $1 - 2y^2 + y^4 + y^4 + 2y^2 - 2y^4 = 1$.

$$1 - 2y^2 + y^4 + y^4 + 2y^2 - 2y^4 = 1$$

Since $x^2 + y^2 = 1$, we have $x^2 = 1 - y^2$ and $y^2 = 1 - x^2$.

$$x^2 = 1 - y^2$$

$$x^2 = 1 - y^2$$

Radio " C_5 " de la esfera tangente a las caras pentagonales de lado " l "

Se obtiene aplicando la fórmula general [2] (ver lám. 33)

$$\begin{aligned} C_5 &= \sqrt{a^2 - (d_5)^2} = \sqrt{\left(\sqrt{\frac{29 + 9\sqrt{5}}{8}} l\right)^2 - \left(\sqrt{\frac{5 + \sqrt{5}}{10}} l\right)^2} = \\ &= \sqrt{\frac{29 + 9\sqrt{5}}{8} - \frac{5 + \sqrt{5}}{10}} \times l = \sqrt{\frac{145 + 45\sqrt{5} - 20 - 4\sqrt{5}}{40}} \times l = \sqrt{\frac{125 + 41\sqrt{5}}{40}} \times l = \\ &= 2,32743844... l \end{aligned}$$

Para el caso del dibujo, será: $C_5 = 2,32743844... \times 22,195 = 51,7 \text{ mm}$

Radio " C_6 " de la esfera tangente a las caras escagonales de lado " l "

Aplicando la fórmula general [2] (ver lám. 33)

$$\begin{aligned} C_6 &= \sqrt{a^2 - (d_6)^2} = \sqrt{\left(\sqrt{\frac{29 + 9\sqrt{5}}{8}} l\right)^2 - l^2} = \sqrt{\frac{29 + 9\sqrt{5}}{8} - 1} \times l = \\ &= \sqrt{\frac{29 + 9\sqrt{5} - 8}{8}} l = \sqrt{\frac{21 + 9\sqrt{5}}{8}} \times l = \sqrt{\frac{3(7 + 3\sqrt{5})}{8}} \times l = \\ &= \sqrt{\frac{3}{8}} \times \sqrt{7 + 3\sqrt{5}} \times l = \sqrt{\frac{3}{8}} \times \left(\sqrt{\frac{9}{2}} + \sqrt{\frac{5}{2}}\right) l = \left(\sqrt{\frac{27}{16}} + \sqrt{\frac{15}{16}}\right) l = \\ &= \left(\frac{3\sqrt{3}}{4} + \frac{\sqrt{15}}{4}\right) l = \frac{3\sqrt{3} + \sqrt{15}}{4} l = 2,26728394... l \end{aligned}$$

Para el caso del dibujo, será: $C_6 = 2,26728394... \times 22,175 = 50,3111$

Ángulo rectilíneo " α_5 " del diedro formado por una cara pentagonal, con el plano diametral del arquimedianos que pasa por una arista de aquella.

Se obtiene, en función de su tangente, por la fórmula general [5] (ver lám. 33).

$$\begin{aligned} \boxed{tg \alpha_5} &= \frac{2 C_5}{\sqrt{4 (d_5)^2 - l^2}} = \frac{2 \sqrt{\frac{125 + 41\sqrt{5}}{40}} l}{\sqrt{4 \left(\sqrt{\frac{5 + \sqrt{5}}{10}} l \right)^2 - l^2}} = \frac{\sqrt{\frac{125 + 41\sqrt{5}}{10}}}{\sqrt{4 \cdot \frac{5 + \sqrt{5}}{10} - 1}} = \\ &= \frac{\sqrt{\frac{125 + 41\sqrt{5}}{10}}}{\sqrt{\frac{20 + 4\sqrt{5}}{10} - 1}} = \frac{\sqrt{\frac{125 + 41\sqrt{5}}{10}}}{\sqrt{\frac{10 + 4\sqrt{5}}{10}}} = \sqrt{\frac{125 + 41\sqrt{5}}{10} : \frac{10 + 4\sqrt{5}}{10}} = \sqrt{\frac{125 + 41\sqrt{5}}{10 + 4\sqrt{5}}} = \\ &= \sqrt{\frac{(125 + 41\sqrt{5})(5 - 2\sqrt{5})}{2 \times (25 - 20)}} = \sqrt{\frac{625 + 205\sqrt{5} - 250\sqrt{5} - 470}{10}} = \sqrt{\frac{215 - 45\sqrt{5}}{10}} = \\ &= \sqrt{\frac{43 - 9\sqrt{5}}{2}} = \frac{\sqrt{43 - 9\sqrt{5}}}{\sqrt{2}} = \frac{\sqrt{\frac{43 + 38}{2}} - \sqrt{\frac{43 - 38}{2}}}{\sqrt{2}} = \frac{\sqrt{\frac{81}{2}} - \sqrt{\frac{5}{2}}}{\sqrt{2}} = \\ &= \sqrt{\frac{81}{4}} - \sqrt{\frac{5}{4}} = \frac{9}{2} - \frac{\sqrt{5}}{2} = \boxed{\frac{9 - \sqrt{5}}{2}} = 3,38196601... \end{aligned}$$

$$tg \alpha_5 = 0,5291692$$

$$\boxed{\alpha_5 = 73^\circ 31' 40,0''}$$

Ángulo rectilíneo " α_6 " del diedro formado por una cara

Handwritten text at the top of the page, possibly a title or header.

Handwritten text in the upper middle section of the page.

Handwritten text in the middle section of the page.

$$\frac{\sqrt{2x+1}}{x^2-1} \cdot \frac{(2x+1)^{1/2}}{(x^2-1)^{1/2}} = \frac{(2x+1)^{3/2}}{(x^2-1)^{3/2}}$$

$$\frac{\sqrt{2x+1}}{\sqrt{x^2-1}} = \frac{(2x+1)^{1/2}}{(x^2-1)^{1/2}}$$

$$\frac{\sqrt{2x+1}}{\sqrt{x^2-1}} = \frac{(2x+1)^{1/2}}{(x^2-1)^{1/2}}$$

$$\frac{\sqrt{2x+1}}{\sqrt{x^2-1}} = \frac{(2x+1)^{1/2}}{(x^2-1)^{1/2}}$$

$$\frac{\sqrt{2x+1}}{\sqrt{x^2-1}} = \frac{(2x+1)^{1/2}}{(x^2-1)^{1/2}}$$

$$\frac{\sqrt{2x+1}}{\sqrt{x^2-1}} = \frac{(2x+1)^{1/2}}{(x^2-1)^{1/2}}$$

Handwritten text at the bottom of the page, possibly a conclusion or footer.

exagonal, con el plano diametral del arquimedianos que pasa por una arista de aquélla.

Se obtiene, en función de su tangente, por la fórmula general [6] (ver lám. 33)

$$\boxed{\tan \alpha_6} = \frac{2 C_6}{\sqrt{4 (d_6)^2 - l^2}} = \frac{2 \times \frac{3\sqrt{3} + \sqrt{5}}{4} l}{\sqrt{4 (l)^2 - l^2}} = \frac{3\sqrt{3} + \sqrt{5}}{2\sqrt{3}} = \frac{9 + \sqrt{45}}{6} =$$

$$= \frac{9 + 3\sqrt{5}}{6} = \boxed{\frac{3 + \sqrt{5}}{2}} = 2, 61803399 \dots$$

$$\tan \alpha_6 = 0, 4177752$$

$$\boxed{\alpha_6 = 69^\circ 5' 41,4''}$$

Ángulo rectilíneo "φ₅₋₆" del diedro formado por una cara pentagonal y otra exagonal, ambas regulares.

Aplicando la fórmula general [4] (ver lám. 33)

$$\boxed{\varphi_{5-6}} = \alpha_5 + \alpha_6 = 73^\circ 31' 40,0'' + 69^\circ 5' 41,4''$$

$$= \boxed{142^\circ 37' 21,4''}$$

También puede obtenerse directamente, así:

$$\boxed{\frac{1}{\tan} \varphi_{5-6}} = \frac{1}{\tan} (\alpha_5 + \alpha_6) = \frac{\frac{1}{\tan} \alpha_5 + \frac{1}{\tan} \alpha_6}{1 - \frac{1}{\tan} \alpha_5 \cdot \frac{1}{\tan} \alpha_6} =$$

$$= \frac{\frac{9 - \sqrt{5}}{2} + \frac{3 + \sqrt{5}}{2}}{1 - \left(\frac{9 - \sqrt{5}}{2}\right) \times \left(\frac{3 + \sqrt{5}}{2}\right)} = \frac{6}{1 - \frac{27 - 3\sqrt{5} + 9\sqrt{5} - 5}{4}} = \frac{6}{1 - \frac{22 + 6\sqrt{5}}{4}} =$$

Let $f(x) = \frac{1}{x^2}$ and $g(x) = \frac{1}{x^3}$. Then $f(x)g(x) = \frac{1}{x^5}$. We want to find the derivative of $f(x)g(x)$ using the product rule.

$$\left(\frac{1}{x^2} \right)' \left(\frac{1}{x^3} \right) + \left(\frac{1}{x^2} \right) \left(\frac{1}{x^3} \right)' = \left(\frac{1}{x^5} \right)' \quad (1)$$

$$-\frac{2}{x^3} \cdot \frac{1}{x^3} + \frac{1}{x^2} \cdot \left(-\frac{3}{x^4} \right) = -\frac{5}{x^5}$$

$$\left(\frac{1}{x^5} \right)' = -\frac{5}{x^6}$$

Now we can find the derivative of $\frac{1}{x^5}$ using the power rule. We have $\frac{1}{x^5} = x^{-5}$. Then $\left(x^{-5} \right)' = -5x^{-6} = -\frac{5}{x^6}$. This matches the result we got using the product rule.

$$\left(\frac{1}{x^5} \right)' = -\frac{5}{x^6}$$

$$\left(\frac{1}{x^5} \right)' = -\frac{5}{x^6} \quad \text{or} \quad \left(x^{-5} \right)' = -5x^{-6}$$

$$\frac{d}{dx} \left(\frac{1}{x^5} \right) = -\frac{5}{x^6}$$

$$= \frac{6}{1 - \frac{11 + 3\sqrt{5}}{2}} = \frac{12}{2 - 11 - 3\sqrt{5}} = -\frac{12}{9 + 3\sqrt{5}} = -\frac{4}{3 + \sqrt{5}} = -\frac{4(3 - \sqrt{5})}{4} =$$

$$= \boxed{-(3 - \sqrt{5})} \quad \text{y haciendo } \alpha_0 = \pi - \varphi_{5-6}, \text{ será:}$$

$$\operatorname{tg} \alpha_0 = -\operatorname{tg} \varphi_{5-6} = -(-(3 - \sqrt{5})) = 3 - \sqrt{5} = 0.76393202\dots$$

$$\operatorname{tg} \alpha_0 = 7.8830553$$

$$\alpha_0 = 37^\circ 22' 38.6''$$

por lo que será:

$$\boxed{\varphi_{5-6}} = 180^\circ - 37^\circ 22' 38.6'' = \boxed{142^\circ 37' 21.4''}$$

valor coincidente con el anterior calculado

Ángulo rectilíneo " φ_{6-6} " del diedro formado por dos caras hexagonales regulares.

Aplicando la fórmula general [4] (ver lám. 33)

$$\boxed{\varphi_{6-6}} = \alpha_6 + \alpha_6 = 2\alpha_6 = 2 \cdot (69^\circ 5' 41.4'') =$$

$$= \boxed{138^\circ 11' 22.8''}$$

También puede obtenerse directamente así:

$$\boxed{\operatorname{tg} \varphi_{6-6}} = \operatorname{tg} 2\alpha_6 = \frac{2 \operatorname{tg} \alpha_6}{1 - \operatorname{tg}^2 \alpha_6} = \frac{2 \times \frac{3 + \sqrt{5}}{2}}{1 - \left(\frac{3 + \sqrt{5}}{2}\right)^2} = \frac{3 + \sqrt{5}}{1 - \frac{9 + 5 + 6\sqrt{5}}{4}} =$$

$$= \frac{3 + \sqrt{5}}{1 - \frac{7 + 3\sqrt{5}}{2}} = \frac{2(3 + \sqrt{5})}{-5 - 3\sqrt{5}} = -\frac{2(3 + \sqrt{5})}{3\sqrt{5} + 5} = -\frac{2(3 + \sqrt{5})(3\sqrt{5} - 5)}{20} =$$

Page 100

Let x and y be any two numbers.

Then $x + y = y + x$.

Also $(x + y) + z = x + (y + z)$.

And $x(yz) = (xy)z$.

Moreover $x(y + z) = xy + xz$.

And $(x + y)z = xz + yz$.

Finally $x(0) = 0$ and $x(1) = x$.

Thus the laws of arithmetic are satisfied.

Therefore the system is a ring.

Q.E.D.

Let R be a ring.

Then R is a group under addition.

Q.E.D.

$$= - \frac{9\sqrt{5} + 15 - 15 - 5\sqrt{5}}{10} = - \frac{4\sqrt{5}}{10} = - \frac{2\sqrt{5}}{5} = - 0,89\ 44\ 27\ 19...$$

y haciendo $\alpha_0 = \pi - \varphi_{6-6}$, será:

$$\tan \alpha_0 = - \tan \varphi_{6-6} = - \left(- \frac{2\sqrt{5}}{5} \right) = \frac{2\sqrt{5}}{5} = 0,89\ 44\ 27\ 19...$$

$$\tan \alpha_0 = 7,95\ 15\ 45\ 0 \qquad \alpha_0 = 41^\circ\ 48'\ 37,2''$$

por lo que será:

$$\varphi_{6-6} = 180^\circ - 41^\circ\ 48'\ 37,2'' = 138^\circ\ 11'\ 22,8''$$

valor coincidente con el anterior calculado

Área lateral "S" del arquimedianos

Se compone de la suma de 12 caras pentagonales y 20 exagonales regulares, todas de lado "l".

La apotema de una cara pentagonal es $\sqrt{\frac{5+\sqrt{5}}{10}} l$, y la de una cara exagonal es $\frac{\sqrt{3}}{2} l$, según se demuestra en Geometría. El área total valdrá pues

$$[S] = 12 \times \frac{5}{2} \times \sqrt{\frac{5+\sqrt{5}}{10}} \times l^2 + 20 \times \frac{6}{2} \times \frac{\sqrt{3}}{2} l^2 = \left(30 \sqrt{\frac{5+\sqrt{5}}{10}} + 30\sqrt{3} \right) l^2 =$$

$$= 30 \left(\sqrt{\frac{5+\sqrt{5}}{10}} + \sqrt{3} \right) l^2 = 77,48\ 10\ 48\ 30... l^2$$

Volumen "V" del arquimedianos

THE UNIVERSITY OF CHICAGO

DEPARTMENT OF THE HISTORY OF ARTS

OFFICE OF THE DEAN

CHICAGO, ILLINOIS

1954

TO THE HONORABLE CHAIRMAN OF THE BOARD OF TRUSTEES

AND THE HONORABLE CHAIRMAN OF THE BOARD OF EDUCATION

OF THE CITY OF CHICAGO

AND THE HONORABLE CHAIRMAN OF THE BOARD OF SUPERVISORS

OF THE COUNTY OF COOK

AND THE HONORABLE CHAIRMAN OF THE BOARD OF COMMISSIONERS

OF THE CITY OF CHICAGO

AND THE HONORABLE CHAIRMAN OF THE BOARD OF ALDERMEN

OF THE CITY OF CHICAGO

AND THE HONORABLE CHAIRMAN OF THE BOARD OF SUPERVISORS

Se compone de la suma de 12 pirámides regulares, de base pentagonal y altura " C_5 ", y de 20 pirámides regulares, de base hexagonal y altura " C_6 ". Su volumen será pues:

$$\begin{aligned}
 \boxed{V} &= 12 \times \frac{5}{2} \times \sqrt{\frac{5+\sqrt{5}}{10}} \ell^2 \times \frac{C_5}{3} + 20 \times \frac{6}{2} \times \frac{\sqrt{3}}{2} \times \ell^2 \times \frac{C_6}{3} = \\
 &= 10 \left(\sqrt{\frac{5+\sqrt{5}}{10}} \times \sqrt{\frac{125+41\sqrt{5}}{40}} + \sqrt{3} \times \frac{3\sqrt{3}+\sqrt{15}}{4} \right) \ell^3 = \\
 &= 10 \left(\sqrt{\frac{(5+\sqrt{5})(125+41\sqrt{5})}{400}} + \frac{9+\sqrt{45}}{4} \right) \ell^3 = 10 \left(\frac{\sqrt{625+125\sqrt{5}+205\sqrt{5}+205}}{20} + \right. \\
 &+ \left. \frac{9+3\sqrt{5}}{4} \right) \ell^3 = 10 \left(\frac{\sqrt{830+330\sqrt{5}}}{20} + \frac{9+3\sqrt{5}}{4} \right) \ell^3 = 10 \left(\frac{\sqrt{10} \times \sqrt{83+33\sqrt{5}}}{20} + \frac{9+3\sqrt{5}}{4} \right) \ell^3 = \\
 &= 10 \left(\frac{\sqrt{10} \times \left(\sqrt{\frac{83+33}{2}} + \sqrt{\frac{83-33}{2}} \right)}{20} + \frac{9+3\sqrt{5}}{4} \right) \ell^3 = 10 \left(\frac{\sqrt{\frac{1210}{2}} + \sqrt{\frac{450}{2}}}{20} + \frac{9+3\sqrt{5}}{4} \right) \ell^3 = \\
 &= 10 \left(\frac{\sqrt{605} + \sqrt{225}}{20} + \frac{9+3\sqrt{5}}{4} \right) \ell^3 = 10 \left(\frac{11\sqrt{5}+15}{20} + \frac{9+3\sqrt{5}}{4} \right) \ell^3 = \\
 &= 10 \left(\frac{11\sqrt{5}+15+45+15\sqrt{5}}{20} \right) \ell^3 = \left(\frac{26\sqrt{5}+60}{2} \right) \ell^3 = \boxed{(13\sqrt{5}+30) \ell^3} = \\
 &= 59,06888371... \ell^3
 \end{aligned}$$

FIGURA CORPÓREA

Se obtiene por acoplamiento de 12 pentágonos regulares de lado " $\ell = 22,2 \text{ mm}$ " y 20 hexágonos regulares de igual lado. El acoplamiento deberá hacerse de forma que en cada vértice concurren un pentágono y dos hexágonos.

The first part of the question is to find the value of x which satisfies the equation $2x + 3 = 7$. This is a simple linear equation and can be solved by subtracting 3 from both sides, giving $2x = 4$, and then dividing by 2, giving $x = 2$.

$$2x + 3 = 7 \quad (1)$$

$$\Rightarrow 2x = 7 - 3$$

$$\Rightarrow 2x = 4$$

$$\Rightarrow x = \frac{4}{2}$$

$$\Rightarrow x = 2$$

$$x = 2$$

Therefore, the value of x is 2.

$$x = 2$$

$$x = 2$$

The second part of the question is to find the value of y which satisfies the equation $3y - 5 = 10$. This is also a simple linear equation and can be solved by adding 5 to both sides, giving $3y = 15$, and then dividing by 3, giving $y = 5$.

En el cuadro sinóptico que damos a continuación, se resumen los resultados analíticos obtenidos anteriormente

CUADRO SINÓPTICO

Magnitud	Valor exacto	Valor decimal aproximado
a	$\sqrt{\frac{29 + 9\sqrt{5}}{8}} \ell$	2, 47 80 19.... ℓ
b	$\frac{3(\sqrt{5} + 1)}{4} \ell$	2, 42 70 51.... ℓ
c_5	$\sqrt{\frac{125 + 47\sqrt{5}}{40}} \ell$	2, 32 74 38.... ℓ
c_6	$\frac{3\sqrt{3} + \sqrt{15}}{4} \ell$	2, 26 72 84.... ℓ
d_5	$\sqrt{\frac{5 + \sqrt{5}}{10}} \ell$	0, 85 06 51.... ℓ
d_6	1 ℓ	1, 00 00 00.... ℓ
m	$3 \cdot \sqrt{\frac{21 + \sqrt{5}}{218}} \ell$	0, 97 94 32... ℓ
α_5	$\operatorname{tg} \alpha_5 = \frac{9 - \sqrt{5}}{2}$	$\operatorname{tg} \alpha_5 = 3.38\ 19\ 66...$ $\alpha_5 = 73^\circ\ 31'\ 40.0''$
α_6	$\operatorname{tg} \alpha_6 = \frac{3 + \sqrt{5}}{2}$	$\operatorname{tg} \alpha_6 = 2.61\ 80\ 34...$ $\alpha_6 = 69^\circ\ 5'\ 41.4''$
φ_{5-6}	$\operatorname{tg} \varphi_{5-6} = -(3 - \sqrt{5})$	$\operatorname{tg} \varphi_{5-6} = -0.76\ 39\ 32$ $\varphi_{5-6} = 142^\circ\ 37'\ 21.4''$
φ_{6-6}	$\operatorname{tg} \varphi_{6-6} = -\frac{2\sqrt{5}}{5}$	$\operatorname{tg} \varphi_{6-6} = -0.89\ 44\ 27...$ $\varphi_{6-6} = 138^\circ\ 11'\ 22.8''$
S	$30 \cdot \left(\sqrt{\frac{5 + \sqrt{5}}{10}} + \sqrt{3} \right) \ell^2$	77, 48 10 48... ℓ^2
V	$(13\sqrt{5} + 30) \ell^3$	59, 06 88 84.... ℓ^3



Received of the Treasurer of the County of ...
the sum of ...

for ...

Date	Particulars	Amount
1890
1891
1892
1893
1894
1895
1896
1897
1898
1899
1900
1901
1902
1903
1904
1905
1906
1907
1908
1909
1910
1911
1912
1913
1914
1915
1916
1917
1918
1919
1920
1921
1922
1923
1924
1925
1926
1927
1928
1929
1930
1931
1932
1933
1934
1935
1936
1937
1938
1939
1940
1941
1942
1943
1944
1945
1946
1947
1948
1949
1950
1951
1952
1953
1954
1955
1956
1957
1958
1959
1960
1961
1962
1963
1964
1965
1966
1967
1968
1969
1970
1971
1972
1973
1974
1975
1976
1977
1978
1979
1980
1981
1982
1983
1984
1985
1986
1987
1988
1989
1990
1991
1992
1993
1994
1995
1996
1997
1998
1999
2000
2001
2002
2003
2004
2005
2006
2007
2008
2009
2010
2011
2012
2013
2014
2015
2016
2017
2018
2019
2020
2021
2022
2023
2024
2025
2026
2027
2028
2029
2030
2031
2032
2033
2034
2035
2036
2037
2038
2039
2040
2041
2042
2043
2044
2045
2046
2047
2048
2049
2050
2051
2052
2053
2054
2055
2056
2057
2058
2059
2060
2061
2062
2063
2064
2065
2066
2067
2068
2069
2070
2071
2072
2073
2074
2075
2076
2077
2078
2079
2080
2081
2082
2083
2084
2085
2086
2087
2088
2089
2090
2091
2092
2093
2094
2095
2096
2097
2098
2099
2100

PROCESO GRÁFICO - ANALÍTICO

Después del cálculo de las magnitudes principales, vamos a proceder en la lámina 45, a la representación gráfica del Arquimediano XIII.

Para su trazado nos valdremos de cotas calculadas por las fórmulas anteriores, de procesos gráficos y de cotas complementarias, cuyo cálculo efectuaremos posteriormente. Todas las magnitudes las obtendremos en función del lado " l_{XIII} " del arquimedianos, cuya longitud es de 22,195 mm.

Con este objeto, calculemos previamente las siguientes magnitudes:

$$l_{XIII} = \text{Dato del ejercicio} = 22,2 \text{ mm}$$

$$a = 2,478019 \times 22,195 = 55,0 \text{ mm}$$

$$b = 2,427051 \times 22,195 = 53,9 \text{ mm}$$

$$C_5 = 2,327438 \times 22,195 = 51,7 \text{ mm}$$

$$C_6 = 2,267284 \times 22,195 = 50,3 \text{ mm}$$

$$d_5 = 0,850651 \times 22,195 = 18,9 \text{ mm}$$

$$d_6 = 1,000000 \times 22,195 = 22,2 \text{ mm}$$

Antes de proceder al trazado gráfico, observemos en la lámina 45, que la proyección del arquimedianos en el plano II, presenta una forma muy regular, debido a la posición elegida en su representación. Esta regularidad nos permite el trazado previo y directo de dicha pro-

[illegible title]

[illegible text]

- [illegible list item 1]
- [illegible list item 2]
- [illegible list item 3]
- [illegible list item 4]
- [illegible list item 5]
- [illegible list item 6]
- [illegible list item 7]

[illegible text]

gección, lo cual nos facilitará la obtención de las proyecciones en II y III como a continuación veremos.

Teniendo presente lo expuesto, el orden de operaciones del trazado gráfico (lámina 45), es el siguiente:

- 1º Situar el centro O, de coordenadas O (72, 72, 85) mm.
- 2º Dibujar en I, II y III las proyecciones de la esfera circunscrita de 55 mm de radio.
- 3º Comenzar el trazado de la proyección II, dividiendo previamente, con gran exactitud, y a partir de A, la circunferencia proyección de la esfera inscrita, en 10 partes iguales.*
- 4º Unir los puntos de división con el centro O.
- 5º Trazar paralelas a ambos lados de los radios anteriores, a distancias iguales a la mitad del lado " l_{XIII} " del arquimediiano. Sobre dichos radios, o sobre sus paralelas se encuentran las proyecciones de "todos" los vértices

*NOTA.- El tomar el punto A como origen de división, tiene por objeto el conseguir que la cara pentagonal superior 1 al 5, la adyacente exagonal 3-9-19-20-10-4 y la exagonal contigua a ésta 19-30-31-32-21-20, ambas en su parte derecha, queden todas perpendiculares a I, con lo cual, los diedros de dichas caras pueden obtenerse directamente en I. Igualmente ocurre esto con su parte simétrica inferior (simetría de centro O).

del arquimedianos. Para determinarlos deberán trazarse circunferencias concéntricas con la exterior y sucesivamente con los siguientes radios:

Vértices	1 al 5	y	56 al 60	Radio	" d_5 "
Vértices	6 al 10	y	51 al 55	Radio	" r_1 "
Vértices	11 al 20	y	41 al 50	Radio	" r_2 "
Vértices	21 al 30	y	31 al 40	Radio	" r_3 "

Los valores analíticos de estos radios los determinaremos posteriormente.

6° Obtenida la proyección total en II del arquimedianos, la determinación de la I y III es inmediata si previamente trazamos en ambas paralelas al eje X, equidistantes del centro O, y a las distancias previamente calculadas " r_1 ", " r_2 " y " r_3 " (cuyos valores determinaremos a continuación), sobre las que se encontrarán las proyecciones de todos sus vértices, en correspondencia con las ya obtenidas en II.

Como comprobación y necesaria ayuda para el trazado gráfico dado anteriormente, vamos a determinar analíticamente las siguientes magnitudes complementarias que darán gran exactitud a dicho trazado.

The first part of the report is devoted to a description of the general situation of the country.

1. General situation	2. Description of the country
3. Description of the population	4. Description of the economy
5. Description of the culture	6. Description of the politics
7. Description of the religion	8. Description of the education
9. Description of the health	10. Description of the environment

The second part of the report is devoted to a description of the specific situation of the country.

The third part of the report is devoted to a description of the specific situation of the country.

The fourth part of the report is devoted to a description of the specific situation of the country.

Apotema " k_6 " de una cara hexagonal

Se demuestra en geometría, es

$$k_6 = \frac{\sqrt{3}}{2} l = 0,8660254... l$$

Para el caso del dibujo, será: $k_6 = 0,8660254... \times 22,195 = 19,2 \text{ mm}$

Distancia " g_1 " de los vértices 6 al 10 al plano de la cara pentagonal 1 al 5, y de los vértices 51 al 55 al de la cara pentagonal 56 al 60.

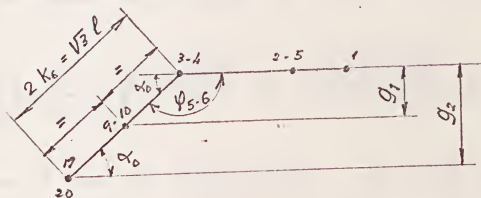


Figura 1

Sea (figura 1) la proyección parcial en I del arquimediano XIII, que comprende la cara pentagonal 1 al 5 y la cantigua hexagonal

(por la parte izquierda) 3-9-19-20-10-4, y que forman entre sí el ángulo conocido ψ_{5-6} , siendo α_0 el ángulo suplementario del mismo.

La altura " g_1 " buscada es la proyección sobre III de la apotema " k_6 " de la cara cantigua hexagonal 3-9-19-20-10-4.

La magnitud será pues:

$$g_1 = k_6 \sen \alpha_0 = \frac{\sqrt{3}}{2} \sen \alpha_0 l \quad [1]$$

THE UNIVERSITY OF CHICAGO
LIBRARY
CHICAGO, ILL.

THE UNIVERSITY OF CHICAGO
LIBRARY
CHICAGO, ILL.

THE UNIVERSITY OF CHICAGO
LIBRARY
CHICAGO, ILL.

pero siendo $\operatorname{tg} \psi_{5.6} = -(3 - \sqrt{5})$, será $\operatorname{tg} \alpha_0 = 3 - \sqrt{5}$

y por lo tanto

$$\begin{aligned} \operatorname{sen} \alpha_0 &= \frac{\operatorname{tg} \alpha_0}{\sqrt{1 + \operatorname{tg}^2 \alpha_0}} = \frac{3 - \sqrt{5}}{\sqrt{1 + (3 - \sqrt{5})^2}} = \frac{3 - \sqrt{5}}{\sqrt{1 + (9 + 5 - 6\sqrt{5})}} = \frac{3 - \sqrt{5}}{\sqrt{15 - 6\sqrt{5}}} = \\ &= \sqrt{\frac{(3 - \sqrt{5})^2}{15 - 6\sqrt{5}}} = \sqrt{\frac{14 - 6\sqrt{5}}{15 - 6\sqrt{5}}} = \sqrt{\frac{2(7 - 3\sqrt{5})}{3(5 - 2\sqrt{5})}} = \sqrt{\frac{2(7 - 3\sqrt{5})(5 + 2\sqrt{5})}{3(25 - 20)}} = \\ &= \sqrt{\frac{2(35 - 15\sqrt{5} + 14\sqrt{5} - 30)}{3 \times 5}} = \sqrt{\frac{2(5 - \sqrt{5})}{15}} \quad [2] \end{aligned}$$

valor que sustituido en [1], nos da

$$\boxed{g_1} = \frac{\sqrt{3}}{2} \times \sqrt{\frac{2(5 - \sqrt{5})}{15}} \cdot l = \sqrt{\frac{6(5 - \sqrt{5})}{4 \times 15}} \cdot l = \boxed{\sqrt{\frac{5 - \sqrt{5}}{10}}} \cdot l = 0,52 \ 57 \ 31 \ 11... l$$

Para el caso del dibujo, será: $g_1 = 0,52 \ 57 \ 31 \ 11... \times 22,195 = 11,7 \text{ m m.}$

Como comprobación numérica del resultado anterior, efectuaremos el siguiente cálculo trigonométrico:

De la fig. 1 se deduce que

$$\begin{aligned} g_1 &= k_6 \cos(\psi_{5.6} - 90^\circ) = \frac{\sqrt{3}}{2} \times \cos(142^\circ 37' 21,4'' - 90^\circ) \times l = \\ &= \frac{\sqrt{3}}{2} \cos 52^\circ 37' 21,4'' \times l \quad \text{y tomando logaritmos} \end{aligned}$$

$$g_1 = 0,52 \ 57 \ 31... l \left\{ \begin{array}{l} \frac{1}{2} \operatorname{lg} 3 = \frac{1}{2} \times 0,477 \ 12 \ 13 = 0,238 \ 56 \ 07 \\ + \operatorname{lg} \cos 52^\circ 37' 21,4'' = 7,783 \ 2 \ 332 \\ 0,02 \ 17 \ 93 \ 9 \\ - \operatorname{lg} 2 = -0,30 \ 10 \ 30 \ 0 \\ \hline \operatorname{lg} 0,52 \ 57 \ 313 = 7,720 \ 7 \ 639 \end{array} \right\}$$

valor muy aproximado al decimal calculado anteriormente.

Distancia "f," entre los dos planos paralelos a Π , que contienen los vértices 6 al 10 y 51 al 55, respectivamente.

Se obtiene por diferencia de las alturas "c₅" y "g₁", ya calculadas.

$$\begin{aligned}
 f_1 &= 2 (c_5 - g_1) = 2 \times \left(\sqrt{\frac{125 + 41\sqrt{5}}{40}} - \sqrt{\frac{5 - \sqrt{5}}{10}} \right) l = \\
 &= 2 \times \left(\sqrt{\left(\sqrt{\frac{125 + 41\sqrt{5}}{40}} - \sqrt{\frac{5 - \sqrt{5}}{10}} \right)^2} \right) \times l = 2 \times \sqrt{\frac{125 + 41\sqrt{5}}{40} + \frac{5 - \sqrt{5}}{10} - 2 \sqrt{\frac{(125 + 41\sqrt{5})(5 - \sqrt{5})}{400}}} \times l = \\
 &= 2 \sqrt{\frac{125 + 41\sqrt{5} + 20 - 4\sqrt{5}}{40} - 2 \times \frac{\sqrt{625 + 205\sqrt{5} - 135\sqrt{5} - 205}}{20}} \times l = \\
 &= 2 \sqrt{\frac{145 + 37\sqrt{5}}{40} - \frac{\sqrt{420 + 80\sqrt{5}}}{10}} \times l = 2 \sqrt{\frac{145 + 37\sqrt{5}}{40} - \frac{\sqrt{20(21 + 4\sqrt{5})}}{10}} \times l \\
 &= 2 \sqrt{\frac{145 + 37\sqrt{5}}{40} - \frac{\sqrt{20} \times \left(\sqrt{\frac{21 + 19}{2}} + \sqrt{\frac{31 - 19}{2}} \right)}{10}} \times l = 2 \sqrt{\frac{145 + 37\sqrt{5}}{40} - \frac{\sqrt{20}(\sqrt{20} + 1)}{10}} \times l = \\
 &= 2 \sqrt{\frac{145 + 37\sqrt{5}}{40} - \frac{20 + 2\sqrt{5}}{10}} \times l = 2 \sqrt{\frac{145 + 37\sqrt{5} - 80 - 8\sqrt{5}}{40}} \times l = 2 \sqrt{\frac{65 + 29\sqrt{5}}{40}} \times l = \\
 &= \sqrt{\frac{65 + 29\sqrt{5}}{10}} \times l = 3,60341465... l
 \end{aligned}$$

Para el caso del dibujo, sea: $f_1 = 3,60341465... \times 22,195 = 80,0 \text{ mm.}$

Radio " r_1 " de las circunferencias que contienen a los
vértices 6 al 10 y 51 al 55 respectivamente

Este radio es un cateto de un triángulo rectángulo de hipotenusa " a " y el otro cateto " $\frac{f_1}{2}$ ". Su valor será:

$$\boxed{r_1} = \sqrt{a^2 - \left(\frac{f_1}{2}\right)^2} = \sqrt{\left(\sqrt{\frac{29 + 9\sqrt{5}}{8}} \ell\right)^2 - \frac{1}{4} \left(\sqrt{\frac{65 + 29\sqrt{5}}{10}} \ell\right)^2} =$$

$$= \sqrt{\frac{29 + 9\sqrt{5}}{8} - \frac{1}{4} \times \frac{65 + 29\sqrt{5}}{10}} \times \ell = \sqrt{\frac{145 + 45\sqrt{5} - 65 - 29\sqrt{5}}{40}} \times \ell = \sqrt{\frac{80 + 16\sqrt{5}}{40}} \ell =$$

$$\boxed{\sqrt{\frac{10 + 2\sqrt{5}}{5}} \times \ell} = 1.70130160 \dots \ell$$

Para el caso del dibujo, será: $r_1 = 1.70130160 \dots \times 22,195 = 37,8 \text{ mm}$

Distancia " g_2 " de los vértices 11 al 20 al plano de la cara
pentagonal 1 al 5, y de los vértices 41 al 50 a la cara pen-
tagonal 56 al 60.

De la figura 1 se deduce que

$$\boxed{g_2} = 2 g_1 = 2 \sqrt{\frac{5 - \sqrt{5}}{10}} \ell = \sqrt{\frac{2(5 - \sqrt{5})}{5}} \times \ell = \boxed{\sqrt{\frac{10 - 2\sqrt{5}}{5}} \times \ell} =$$

$$= 1.05146222 \dots \ell$$

Para el caso del dibujo, será: $g_2 = 1.05146222 \dots \times 22,195 = 23,3$

I am very happy to hear that you are well and hope you are enjoying your studies. I am also well and hope you are enjoying your studies. I am also well and hope you are enjoying your studies.

Yours faithfully,
[Signature]

I am very happy to hear that you are well and hope you are enjoying your studies. I am also well and hope you are enjoying your studies. I am also well and hope you are enjoying your studies.

Yours faithfully,
[Signature]

I am very happy to hear that you are well and hope you are enjoying your studies. I am also well and hope you are enjoying your studies. I am also well and hope you are enjoying your studies.

I am very happy to hear that you are well and hope you are enjoying your studies. I am also well and hope you are enjoying your studies. I am also well and hope you are enjoying your studies.

Yours faithfully,
[Signature]

I am very happy to hear that you are well and hope you are enjoying your studies. I am also well and hope you are enjoying your studies. I am also well and hope you are enjoying your studies.

Yours faithfully,
[Signature]

I am very happy to hear that you are well and hope you are enjoying your studies. I am also well and hope you are enjoying your studies. I am also well and hope you are enjoying your studies.

I am very happy to hear that you are well and hope you are enjoying your studies. I am also well and hope you are enjoying your studies. I am also well and hope you are enjoying your studies.

Distancia "f₂" entre los dos planos paralelos a H que contienen los vértices 11 al 20 y 41 al 50 respectivamente.

Se obtiene por diferencia de las alturas "c₅" y "g₂", ya calculadas.

$$\begin{aligned}
 f_2 &= 2 (c_5 - g_2) = 2 \left(\sqrt{\frac{125 + 41\sqrt{5}}{40}} - \sqrt{\frac{10 - 2\sqrt{5}}{5}} \right) \times l = \\
 &= 2 \times \sqrt{\left(\sqrt{\frac{125 + 41\sqrt{5}}{40}} - \sqrt{\frac{10 - 2\sqrt{5}}{5}} \right)^2} \times l = 2 \sqrt{\frac{125 + 41\sqrt{5}}{40} + \frac{10 - 2\sqrt{5}}{5} - 2 \sqrt{\frac{(125 + 41\sqrt{5})(10 - 2\sqrt{5})}{200}}} \times l = \\
 &= 2 \sqrt{\frac{125 + 41\sqrt{5} + 80 - 16\sqrt{5}}{40} - 2 \sqrt{\frac{2 \times (625 + 205\sqrt{5} - 125\sqrt{5} - 205)}{200}}} \times l = \\
 &= 2 \sqrt{\frac{205 + 25\sqrt{5}}{40} - \frac{2}{10} \sqrt{420 + 80\sqrt{5}}} \times l = 2 \times \sqrt{\frac{41 + 5\sqrt{5}}{8} - \frac{1}{5} \sqrt{20(21 + 4\sqrt{5})}} \times l = \\
 &= 2 \left(\sqrt{\frac{41 + 5\sqrt{5}}{8}} - \frac{1}{5} \sqrt{20} \left(\sqrt{\frac{21 + 19}{2}} + \sqrt{\frac{21 - 19}{2}} \right) \right) \times l = 2 \left(\sqrt{\frac{41 + 5\sqrt{5}}{8}} - \frac{\sqrt{20}(\sqrt{20} + 1)}{5} \right) l = \\
 &= 2 \left(\sqrt{\frac{41 + 5\sqrt{5}}{8}} - \frac{20 + 2\sqrt{5}}{5} \right) \times l = 2 \sqrt{\frac{205 + 25\sqrt{5} - 160 - 16\sqrt{5}}{40}} \times l = \sqrt{\frac{45 + 9\sqrt{5}}{10}} l = \\
 &= \sqrt{\frac{9(5 + \sqrt{5})}{10}} l = \boxed{3 \sqrt{\frac{5 + \sqrt{5}}{10}} l} = 3 d_5 = 2, 55 \ 19 \ 52 \ 64 \dots l
 \end{aligned}$$

El valor en el dibujo será: f₂ = 2,55 19 52 64 ... × 22,195 = 56,6 mm

Radio "r₂" de las circunferencias que contienen a los vértices 11 al 20 y 41 al 50 respectivamente

Este radio es un cateto de un triángulo rectángulo

$f(x) = \int_0^x \frac{1}{1+t^2} dt$. It is well known that this function is increasing and concave down on the interval $(-\infty, \infty)$. Moreover, it is easy to see that $f(x) \rightarrow 0$ as $x \rightarrow -\infty$ and $f(x) \rightarrow \frac{\pi}{2}$ as $x \rightarrow \infty$.

$$\lim_{x \rightarrow \infty} f(x) = \frac{\pi}{2} \quad (1)$$

It is also known that the function $f(x)$ is invertible on the interval $(-\infty, \infty)$ and its inverse function is denoted by $g(y)$.

$$g(y) = \tan\left(\frac{y}{2}\right) \quad (2)$$

It is easy to see that the function $g(y)$ is increasing and concave up on the interval $(-\frac{\pi}{2}, \frac{\pi}{2})$.

Moreover, it is easy to see that $g(y) \rightarrow -\infty$ as $y \rightarrow -\frac{\pi}{2}^+$ and $g(y) \rightarrow \infty$ as $y \rightarrow \frac{\pi}{2}^-$.

It is also known that the function $g(y)$ is invertible on the interval $(-\frac{\pi}{2}, \frac{\pi}{2})$ and its inverse function is denoted by $h(x)$.

$$h(x) = 2 \arctan(x) \quad (3)$$

It is easy to see that the function $h(x)$ is increasing and concave down on the interval $(-\infty, \infty)$.

Moreover, it is easy to see that $h(x) \rightarrow -\frac{\pi}{2}$ as $x \rightarrow -\infty$ and $h(x) \rightarrow \frac{\pi}{2}$ as $x \rightarrow \infty$.

It is also known that the function $h(x)$ is invertible on the interval $(-\infty, \infty)$ and its inverse function is denoted by $k(y)$.

de hipotenusa "a" y el otro cateto " $\frac{f_2}{2}$ ". Su valor será:

$$\begin{aligned} \boxed{r_2} &= \sqrt{a^2 - \left(\frac{f_2}{2}\right)^2} = \sqrt{\left(\sqrt{\frac{29 + 9\sqrt{5}}{8}} l\right)^2 - \frac{1}{4} \left(3\sqrt{\frac{5 + \sqrt{5}}{10}} l\right)^2} = \\ &= \sqrt{\frac{29 + 9\sqrt{5}}{8} - \frac{9}{4} \times \frac{5 + \sqrt{5}}{10}} \times l = \sqrt{\frac{29 + 9\sqrt{5}}{8} - \frac{45 + 9\sqrt{5}}{40}} \times l = \\ &= \sqrt{\frac{145 + 45\sqrt{5} - 45 - 9\sqrt{5}}{40}} \times l = \sqrt{\frac{100 + 36\sqrt{5}}{40}} \times l = \boxed{\sqrt{\frac{25 + 9\sqrt{5}}{10}}} \times l = \\ &= 2, 12 \ 42 \ 55 \ 44 \dots l \end{aligned}$$

Para el caso del dibujo, será: $r_2 = 2, 12 \ 42 \ 55 \ 44 \dots \times 22, 195 = 46,7 \text{ mm}$

Distancia "g₂" de los vértices 21 al 30 al plano de la cara pentagonal 1 al 5, y de los vértices 31 al 40 al de la cara pentagonal 56 al 60

Refiriéndonos a la lámina 45, vemos que la cara exagonal : 19 - 30 - 31 - 32 - 21 - 20, contigua a la exagonal 3-9-19-20-10-4 y de arista común 19-20, están las dos proyectadas sobre I según líneas rectas, por ser sus respectivos planos perpendiculares a I, y por lo tanto la arista común 19-20, intersección de dichas caras, será también perpendicular a I.

En la figura 2 representamos el contorno superior izquierdo del arquimediante en dicha zona, que in-

1850

1850

1850

1850

1850

1850

1850

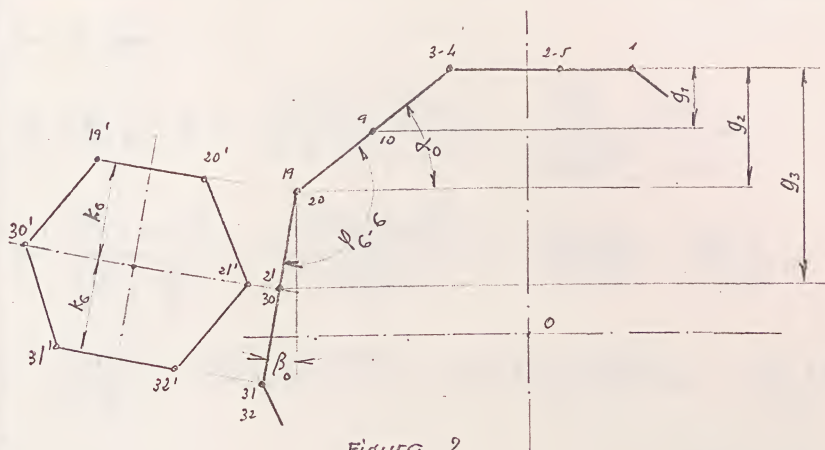


Figura 2

cluye la representada en la figura 1. La cara exagonal 3-7-19-20-10-4, tiene contigua la decagonal superior 1 al 5, paralela a II, y la también exagonal 19-30-31-32-21-20, oblicua a II; esta última la hemos representado también abatida sobre el plano del dibujo (parte izquierda de la figura).

De la figura se deduce:

$$\varphi_{6-6} - \alpha_0 = \frac{\pi}{2} + \beta_0 \quad [1]$$

siendo " β_0 " el ángulo de proyección sobre III de la mencionada cara decagonal oblicua inferior.

De la [1] se deduce:

$$\operatorname{tg} (\varphi_{6-6} - \alpha_0) = \operatorname{tg} \left(\frac{\pi}{2} + \beta_0 \right) = -\operatorname{ctg} \beta_0 \quad [2]$$

pero ya hemos deducido en el cálculo de " g_1 " (ver pag. 17) que

$$\operatorname{tg} \alpha_0 = 3 - \sqrt{5} \quad \text{y que (ver pag. 10)} \quad \operatorname{tg} \varphi_{6-6} = -\frac{2\sqrt{5}}{5}$$



The first part of the sketch shows a large, irregular shape that could be a building or a landscape feature. To the right of this shape, there is a more defined, geometric structure that resembles a bridge or a large, open-sided building. The drawing is done in a simple, sketchy style with light lines.

The second part of the sketch, located below the first, shows a similar but more complex structure. It appears to be a bridge or a large building with multiple sections and a more intricate design. The lines are still light and sketchy, but the overall form is more detailed than the first part.

The third part of the sketch, at the bottom, shows a structure that looks like a bridge or a large building with a series of arches or sections. The drawing is very light and sketchy, with the lines barely visible against the background.

por lo que

$$\begin{aligned} \operatorname{tg} (\varphi_{6-6} - \alpha_0) &= \frac{\operatorname{tg} \varphi_{6-6} - \operatorname{tg} \alpha_0}{1 + \operatorname{tg} \varphi_{6-6} \times \operatorname{tg} \alpha_0} = \frac{-\frac{2\sqrt{5}}{5} - (3 - \sqrt{5})}{1 + \left(-\frac{2\sqrt{5}}{5}\right) \times (3 - \sqrt{5})} = \\ &= \frac{-\frac{2\sqrt{5}}{5} - 3 + \sqrt{5}}{1 - \frac{6\sqrt{5}}{5} + \frac{10}{5}} = \frac{-\frac{2\sqrt{5}}{5} - \frac{15}{5} + \frac{5\sqrt{5}}{5}}{\frac{5 - 6\sqrt{5} + 10}{5}} = \frac{3\sqrt{5} - 15}{15 - 6\sqrt{5}} = \frac{\sqrt{5} - 5}{5 - 2\sqrt{5}} = \\ &= -\frac{5 - \sqrt{5}}{5 - 2\sqrt{5}} = -\frac{(5 - \sqrt{5})(5 + 2\sqrt{5})}{5} = -\frac{25 - 5\sqrt{5} + 10\sqrt{5} - 10}{5} = -\frac{15 + 5\sqrt{5}}{5} = \\ &= -(3 + \sqrt{5}) \quad \text{valor que sustituido en [2] nos da} \end{aligned}$$

$$-(3 + \sqrt{5}) = -\operatorname{ctg} \beta_0 \quad " \quad \operatorname{ctg} \beta_0 = 3 + \sqrt{5} \quad "$$

$$\operatorname{tg} \beta_0 = \frac{1}{3 + \sqrt{5}} = \frac{3 - \sqrt{5}}{4} \quad \text{de esta última} \quad [3]$$

$$\begin{aligned} \boxed{\cos \beta_0} &= \frac{1}{\sqrt{1 + \operatorname{tg}^2 \beta_0}} = \frac{1}{\sqrt{1 + \left(\frac{3 - \sqrt{5}}{4}\right)^2}} = \frac{1}{\sqrt{1 + \frac{9 + 5 - 6\sqrt{5}}{16}}} = \frac{1}{\sqrt{1 + \frac{14 - 6\sqrt{5}}{16}}} = \\ &= \frac{1}{\sqrt{1 + \frac{7 - 3\sqrt{5}}{8}}} = \frac{1}{\sqrt{\frac{8 + 7 - 3\sqrt{5}}{8}}} = \sqrt{\frac{8}{15 - 3\sqrt{5}}} = \sqrt{\frac{8}{3(5 - \sqrt{5})}} = \sqrt{\frac{8(5 + \sqrt{5})}{3 \times 20}} = \end{aligned}$$

$$\boxed{\sqrt{\frac{2(5 + \sqrt{5})}{15}}} \quad \text{de la fig. 2 se deduce finalmente que} \quad [4]$$

$$\begin{aligned} \boxed{g_3} &= g_2 + k_6 \Leftrightarrow \beta_0 = \sqrt{\frac{10 - 2\sqrt{5}}{5}} \cdot l + \frac{\sqrt{3}}{2} l \times \sqrt{\frac{2(5 + \sqrt{5})}{15}} = \\ &= \left(\sqrt{\frac{10 - 2\sqrt{5}}{5}} + \sqrt{\frac{6(5 + \sqrt{5})}{60}} \right) l = \left(\sqrt{\frac{10 - 2\sqrt{5}}{5}} + \sqrt{\frac{5 + \sqrt{5}}{10}} \right) l = \end{aligned}$$

First line of the main handwritten text.

Second line of the main handwritten text.

Third line of the main handwritten text.

Fourth line of the main handwritten text.

Fifth line of the main handwritten text.

Sixth line of the main handwritten text.

Seventh line of the main handwritten text.

Eighth line of the main handwritten text.

Ninth line of the main handwritten text.

Tenth line of the main handwritten text.

Eleventh line of the main handwritten text.

$$\begin{aligned}
 &= \sqrt{\left(\sqrt{\frac{10-2\sqrt{5}}{5}} + \sqrt{\frac{5+\sqrt{5}}{10}}\right)^2} \times l = \sqrt{\frac{10-2\sqrt{5}}{5} + \frac{5+\sqrt{5}}{10} + 2\sqrt{\frac{(10-2\sqrt{5})(5+\sqrt{5})}{50}}} \times l = \\
 &= \sqrt{\frac{20-4\sqrt{5}+5+\sqrt{5}}{10} + 2\sqrt{\frac{50-10\sqrt{5}+10\sqrt{5}-10}{50}}} \times l = \sqrt{\frac{25-3\sqrt{5}}{10} + 2\sqrt{\frac{40}{50}}} \times l = \\
 &= \sqrt{\frac{25-3\sqrt{5}}{10} + 2 \times \frac{2}{\sqrt{5}}} \times l = \sqrt{\frac{25-3\sqrt{5}}{10} + \frac{4\sqrt{5}}{5}} \times l = \sqrt{\frac{25-3\sqrt{5}+8\sqrt{5}}{10}} \times l = \\
 &= \sqrt{\frac{25+5\sqrt{5}}{10}} \times l = \boxed{\sqrt{\frac{5+\sqrt{5}}{2}}} \times l = 1,90 \ 21 \ 13 \ 03 \dots l
 \end{aligned}$$

Para el caso del dibujo, será: $g_3 = 1,90 \ 21 \ 13 \ 03 \dots \times 22,195 = 42,2 \text{ mm}$

Distancia "f₃" entre los dos planos paralelos a II que contienen los vértices 21 al 30 y 31 al 40 respectivamente

Se obtiene por diferencia de las alturas "c₅" y "g₃" ya calculadas

$$\begin{aligned}
 \boxed{f_3} &= 2(c_5 - g_3) = 2\left(\sqrt{\frac{125+41\sqrt{5}}{40}} - \sqrt{\frac{5+\sqrt{5}}{2}}\right) l = \\
 &= 2\sqrt{\left(\sqrt{\frac{125+41\sqrt{5}}{40}} - \sqrt{\frac{5+\sqrt{5}}{2}}\right)^2} l = 2\sqrt{\frac{125+41\sqrt{5}}{40} + \frac{5+\sqrt{5}}{2} - 2\sqrt{\frac{(125+41\sqrt{5})(5+\sqrt{5})}{80}}} l = \\
 &= 2\sqrt{\frac{125+41\sqrt{5}+100+20\sqrt{5}}{40} - \frac{\sqrt{625+205\sqrt{5}+125\sqrt{5}+205}}{20}} \times l = \\
 &= 2\sqrt{\frac{225+61\sqrt{5}}{40} - \frac{\sqrt{830+330\sqrt{5}}}{20}} \times l = 2\sqrt{\frac{225+61\sqrt{5}}{40} - \frac{\sqrt{83+33\sqrt{5}}}{\sqrt{2}}} l = \\
 &= 2\sqrt{\frac{225+61\sqrt{5}}{40} - \frac{\sqrt{\frac{83+38}{2}} + \sqrt{\frac{83-38}{2}}}{\sqrt{2}}} \times l = 2\sqrt{\frac{225+61\sqrt{5}}{40} - \frac{\sqrt{121}}{4} - \frac{\sqrt{45}}{4}} \times l =
 \end{aligned}$$

THE UNIVERSITY OF CHICAGO PRESS
1215 EAST 58TH STREET
CHICAGO, ILLINOIS 60637
TEL: 773-707-3000
FAX: 773-707-3000
WWW.CHICAGO.PRESS.EDU

$$= 2 \sqrt{\frac{225 + 61\sqrt{5}}{40} - \frac{11}{2} - \frac{3\sqrt{5}}{2}} \times l = 2 \sqrt{\frac{225 + 61\sqrt{5} - 220 - 60\sqrt{5}}{40}} \times l =$$

$$\boxed{\sqrt{\frac{5 + \sqrt{5}}{10}}} l = 0,85\ 06\ 50\ 88... l$$

Para el caso del dibujo, será: $f_3 = 0,85\ 06\ 50\ 88... \times 22,195 = 18,9\ mm$

Radio "r₃" de las circunferencias que contienen a los
vértices 21 al 30 y 31 al 40 respectivamente

Este radio es un cateto de un triángulo rectángulo de
hipotenusa "a" y el otro cateto " $\frac{f_3}{2}$ ". Su valor será:

$$\boxed{r_3} = \sqrt{a^2 - \left(\frac{f_3}{2}\right)^2} = \sqrt{\left(\sqrt{\frac{29 + 9\sqrt{5}}{8}} l\right)^2 - \frac{1}{4} \left(\sqrt{\frac{5 + \sqrt{5}}{10}} l\right)^2} =$$

$$= \sqrt{\frac{29 + 9\sqrt{5}}{8} - \frac{1}{4} \times \frac{5 + \sqrt{5}}{10}} \times l = \sqrt{\frac{145 + 45\sqrt{5} - 5 - \sqrt{5}}{40}} \times l = \sqrt{\frac{140 + 44\sqrt{5}}{40}} \times l =$$

$$\boxed{\sqrt{\frac{35 + 11\sqrt{5}}{10}}} l = 2,44\ 12\ 44\ 51... l$$

Para el caso del dibujo, será: $r_3 = 2,44\ 12\ 44\ 51... \times 22,195 = 54,2\ mm$

En el cuadro sinóptico que damos a continuación, re-
sumimos los resultados de los valores complementarios
deducidos.

The first part of the paper is devoted to the study of the properties of the function $f(x)$ defined by the equation

$$f(x) = \int_0^x \frac{1}{1+t^2} dt$$

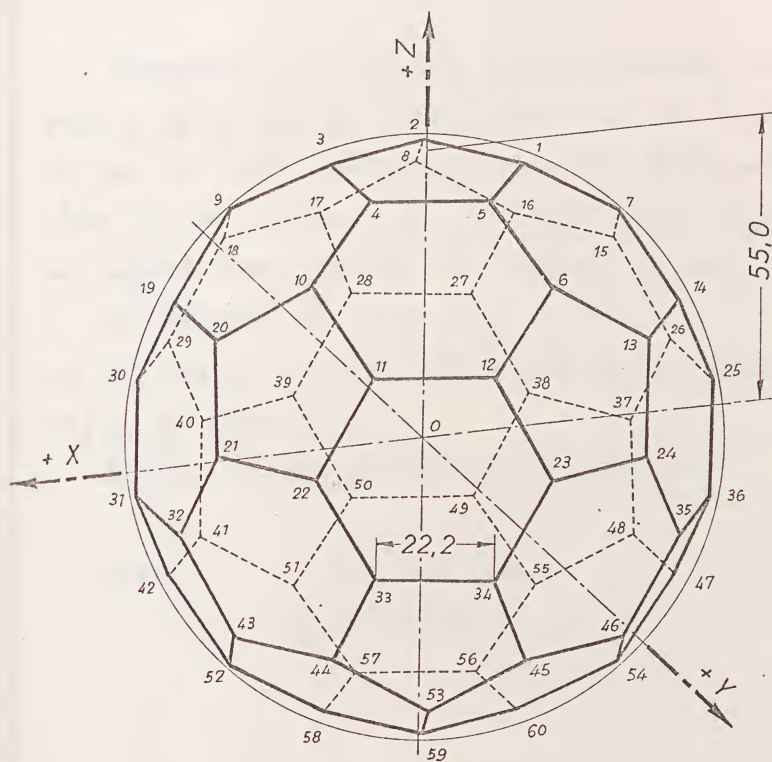
It is well known that this function is the arctangent function, i.e. $f(x) = \arctan x$. The main result of this section is the proof of the following theorem:

Theorem 1. Let $f(x)$ be the function defined by the equation (1). Then for any x we have

$$f(x) = \frac{1}{2} \pi - \arctan \frac{1}{x}$$

CUADRO SINÓPTICO DE LAS MAGNITUDES COMPLEMENTARIAS

Magnitud	Valor exacto	Valor decimal aproximado
k_5	$\sqrt{\frac{5 + 2\sqrt{5}}{20}} \ell$	0, 68 81 91... ℓ
k_6	$\frac{\sqrt{3}}{2} \ell$	0, 86 60 25... ℓ
f_1	$\sqrt{\frac{65 + 29\sqrt{5}}{10}} \ell$	3, 60 34 15... ℓ
f_2	$3 \sqrt{\frac{5 + \sqrt{5}}{10}} \ell$	2, 55 19 53... ℓ
f_3	$\sqrt{\frac{5 + \sqrt{5}}{10}} \ell$	0, 85 06 51... ℓ
g_1	$\sqrt{\frac{5 - \sqrt{5}}{10}} \ell$	0, 52 57 31... ℓ
g_2	$\sqrt{\frac{10 - 2\sqrt{5}}{5}} \ell$	1, 05 14 62... ℓ
g_3	$\sqrt{\frac{5 + \sqrt{5}}{2}} \ell$	1, 90 21 13... ℓ
r_1	$\sqrt{\frac{10 + 2\sqrt{5}}{5}} \ell$	1, 70 13 02... ℓ
r_2	$\sqrt{\frac{25 + 9\sqrt{5}}{10}} \ell$	2, 12 42 55... ℓ
r_3	$\sqrt{\frac{35 + 11\sqrt{5}}{10}} \ell$	2, 44 12 45... ℓ



Arquimediano XIII



THE END OF THE WORLD

ENUNCIADO

Representar, por el método gráfico-analítico, en los planos I, II y III, un Arquimediano de la Serie A_n , en el que en cada vértice concurren tres triángulos equiláteros y un polígono de " n " lados, todos regulares y de igual lado, siendo " n " natural y mayor que 3

La longitud del lado, para $n = 7$ es de 44.7 mm, y las coordenadas de su centro O, son: O (72, 72, 85) mm.

Dibujar en formato A3V y a escala 1:1

DATOS: O (72, 72, 85) mm

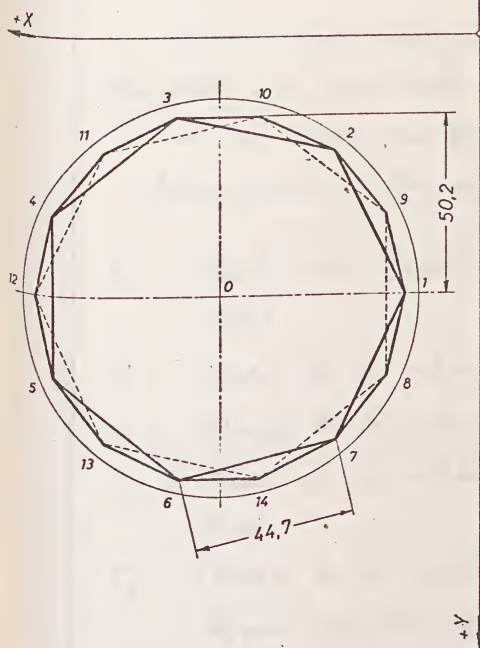
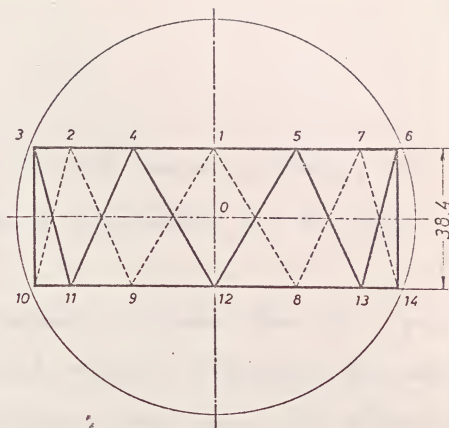
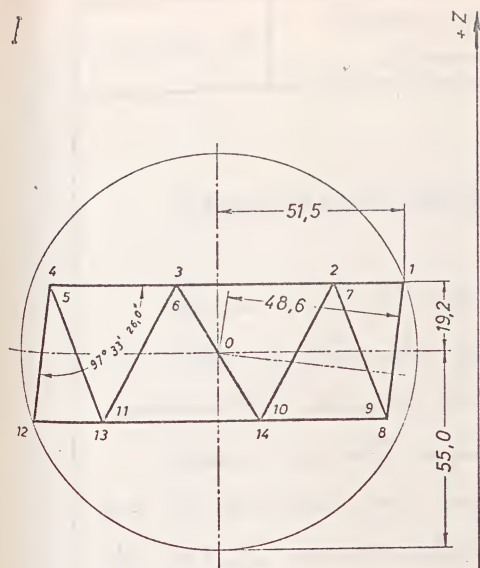
$$l_{27} = 44.7 \text{ mm}$$

The first of the three parts of the book is devoted to a general introduction to the subject of the book. The second part is devoted to a detailed description of the various methods of the book. The third part is devoted to a detailed description of the various methods of the book.

The first of the three parts of the book is devoted to a general introduction to the subject of the book. The second part is devoted to a detailed description of the various methods of the book. The third part is devoted to a detailed description of the various methods of the book.

The first of the three parts of the book is devoted to a general introduction to the subject of the book. The second part is devoted to a detailed description of the various methods of the book. The third part is devoted to a detailed description of the various methods of the book.

The first of the three parts of the book is devoted to a general introduction to the subject of the book. The second part is devoted to a detailed description of the various methods of the book. The third part is devoted to a detailed description of the various methods of the book.



ARQUIMEDIANOS Serie A_n

Número de caras triangulares.....	$C_3 = 2n$
Número de caras regulares de "n" lados..	$C_n = 2$
Número de vértices.....	$V = 2n$
Número de aristas.....	$A = 4n$
Número de caras de un ángulo sólido...	$3P_3 + 1P_n$

ENUNCIADO

Representar por el método gráfico-analítico, en los planos I, II y III, un Arquimediano de la Serie A_n , en el que en cada vértice concurren tres triángulos equiláteros y un polígono regular de "n" lados, todos de igual longitud, siendo "n" un número natural mayor que 3. La longitud del lado, para $n = 7$, es de 44,7 mm, y las coordenadas de su centro son: O (72, 72, 85).

Dibujar en formato A3v y a escala 1:1.

Propuesta	De entrega	Entregada	Calificación	(firma)	Escuela
Fecha:					Curso
Alumno:					
Escala	Arquimedianos Serie A_n				Lámina 46
1:1					Curso 19 - 15



Notes on the diagram

- 1. The diagram shows a circle with a horizontal diameter and a vertical radius.
- 2. The curved lines inside the circle represent a stylized 'W' or a series of connected arches.
- 3. The drawing is very light and appears to be a preliminary sketch.

Conclusion

The diagram illustrates the relationship between the circle and the curved lines. The curved lines are drawn such that they touch the circle at several points, creating a series of arches. This suggests a geometric construction or a specific mathematical property. The drawing is very light and appears to be a preliminary sketch.



21-5

in the center of the circle

CONSIDERACIONES PREVIAS

Exigiremos en el estudio de estos arquimedianos, las directrices y fórmulas generales planteadas en el "Arquimedianos I", lámina 33.

Existen infinito arquimedianos de esta Serie A_n , ya que el número de lados del polígono regular P_n , puede ser un número natural cualquiera mayor que 3. El estudio que realizamos a continuación considera este polígono en general, para cualquier valor de " n ", así como la longitud de su lado correspondiente que designaremos a su vez, en forma general " l_n " ($n > 3$).

Determinaremos las siguientes magnitudes

l_n = Arista del Arquimedianos Serie A_n (dato del ejercicio).

a = Radio de la esfera circunscrita

b = Radio de la esfera tangente a las aristas

c_3 = Radio de la esfera tangente a las caras triangulares.

c_n = Radio de la esfera tangente a las caras del polígono regular de " n " lados que en todos los casos son tan sólo dos P_n .

d_3 = Radio de la circunferencia circunscrita a una cara triangular

- d_n = Radio de la circunferencia circunscrita a una cara regular de "n" lados.
- m = Radio de la circunferencia circunscrita al polígono obtenido al unir los extremos de las aristas de un ángulo sólido.
- α_3 = Ángulo rectilíneo del diedro formado por una cara triangular, con el plano diametral del arquimedeano que pasa por una arista de aquella.
- α_n = Ángulo rectilíneo del diedro formado por una cara regular de "n" lados, con el plano diametral del arquimedeano que pasa por una arista de aquella.
- φ_{3-n} = Ángulo rectilíneo del diedro formado por una cara triangular y otra regular de "n" lados.
- φ_{3-3} = Ángulo rectilíneo del diedro formado por dos caras triangulares.
- S = Superficie
- V = Volumen

PROCESO GRÁFICO-ANALÍTICO

El estudio realizado de este arquimedeano, nos indica que se compone de " $2n$ " caras triangulares, 2 caras regulares de "n" lados; " $2n$ " vértices y " $4n$ " aristas.

En cada vértice concurren 3 caras triangulares y una regular de "n" lados, todas de igual lado.

By the Hon. the Secretary

Published by the Royal Anthropological Institute, 21, BEDFORD SQUARE, LONDON, W.C.

—

Printed by the Royal Anthropological Institute, 21, BEDFORD SQUARE, LONDON, W.C.

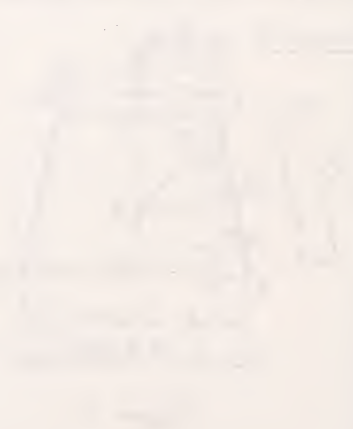
Published by the Royal Anthropological Institute, 21, BEDFORD SQUARE, LONDON, W.C.

London: 1880.

Price 1s.

Published by the Royal Anthropological Institute, 21, BEDFORD SQUARE, LONDON, W.C.

THE JOURNAL OF THE
ROYAL ANTHROPOLOGICAL INSTITUTE
Vol. 10
Part 1
1880
Published by the Royal Anthropological Institute, 21, BEDFORD SQUARE, LONDON, W.C.



Published by the Royal Anthropological Institute, 21, BEDFORD SQUARE, LONDON, W.C.

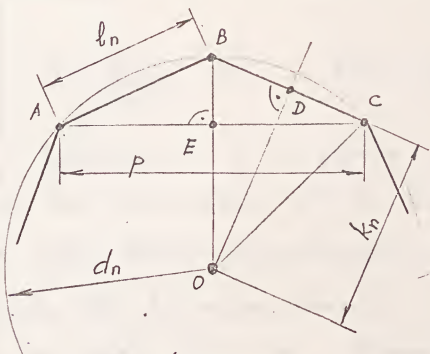


Figura 2

En dicha figura 2, tenemos $\overline{AB} = \overline{BC} = l_n$, dos lados consecutivos de un polígono regular de "n" lados, siendo " l_n " la longitud de su lado, y O el centro de su circunferencia circunscrita.

Unamos los extremos \overline{B} y \overline{C} de un lado, con el centro O; el triángulo \overline{OBC} será isósceles y su altura OD, perpendicular a \overline{BC} en su punto medio y bisectriz del ángulo \widehat{BOC} .

De la figura se deduce:

$$\widehat{BOC} = \frac{2\pi}{n} \quad " \quad \widehat{BOD} = \frac{\widehat{BOC}}{2} = \frac{\pi}{n} \quad " \quad \widehat{DBO} = \frac{\pi}{2} - \frac{\pi}{n}$$

$$\overline{EC} = \overline{BC} \operatorname{sen} \widehat{DBO} = l_n \operatorname{sen} \left(\frac{\pi}{2} - \frac{\pi}{n} \right) = l_n \cos \frac{\pi}{n}$$

y finalmente

$$\overline{AC} = \boxed{P} = 2 \cdot \overline{EC} = 2 l_n \cos \frac{\pi}{n}$$

[1]

La apotema del polígono será:

$$\text{apotema} = \overline{OD} = \boxed{kn} = \overline{BD} \times \operatorname{tg} \widehat{DBO} = \frac{l_n}{2} : \operatorname{tg} \frac{\pi}{n}$$

[2]

y el radio de la circunferencia circunscrita:



The first part of the paper is devoted to the study of the properties of the function $f(x)$ defined on the interval $[0, 1]$. It is shown that $f(x)$ is continuous and differentiable on this interval. The derivative of $f(x)$ is given by the formula $f'(x) = \dots$.



In the second part of the paper, we consider the problem of finding the maximum and minimum values of the function $f(x)$ on the interval $[0, 1]$. It is shown that the maximum value of $f(x)$ is attained at $x = \dots$ and the minimum value is attained at $x = \dots$.

$$f(0) = 0, f(1) = 0, f'(x) = \dots$$

$$f''(x) = \dots$$

Finally, we consider the problem of finding the area under the curve $y = f(x)$ from $x = 0$ to $x = 1$. It is shown that the area is given by the formula $A = \dots$.

$$A = \int_0^1 f(x) dx = \dots$$

The results of the calculations are summarized in the following table:

$$\overline{OB} = \boxed{d_n} = \overline{BD} : \cos \widehat{DBO} = \frac{l_n}{2} : \cos \left(\frac{\pi}{2} - \frac{\pi}{n} \right) = \boxed{\frac{l_n}{2} : \operatorname{sen} \frac{\pi}{n}} \quad [3]$$

Refiriéndonos a la figura 1, tracemos por \overline{E} y \overline{F} , puntos medios respectivos de los lados \overline{BC} y \overline{CD} , perpendiculares a estos, dichas perpendiculares se cortarán en un punto O , centro de la circunferencia circunscrita al trapecio $\overline{A-B-C-D}$, y de radio $\overline{OC} = m$. Trazando seguidamente por \overline{C} la perpendicular a \overline{BA} , se nos formará el triángulo rectángulo \overline{CBG} , recto en G ; en éste se verificará que

$$\overline{BG} = \frac{\overline{BA} - \overline{CD}}{2} = \frac{2 l_n \cos \frac{\pi}{n} - l_n}{2} = l_n \cos \frac{\pi}{n} - \frac{l_n}{2} = \left(\cos \frac{\pi}{n} - \frac{1}{2} \right) l_n$$

pero siendo

$$\cos \widehat{CBG} = \operatorname{sen} \widehat{BCG} = \frac{BG}{BC} = \frac{\left(\cos \frac{\pi}{n} - \frac{1}{2} \right) l_n}{l_n} = \cos \frac{\pi}{n} - \frac{1}{2}$$

y también

$$\alpha = 2\pi - \widehat{CBG}$$

$$\text{será} \quad \cos \alpha = \cos (2\pi - \widehat{CBG}) = -\cos \widehat{CBG} = -\left(\cos \frac{\pi}{n} - \frac{1}{2} \right)$$

por lo que

$$\cos \frac{\alpha}{2} = \sqrt{\frac{1}{2} (1 + \cos \alpha)} = \sqrt{\frac{1 - \left(\cos \frac{\pi}{n} - \frac{1}{2} \right)}{2}} = \sqrt{\frac{\frac{3}{2} - \cos \frac{\pi}{n}}{2}} = \sqrt{\frac{3 - 2 \cos \frac{\pi}{n}}{4}}$$

y finalmente, de la figura 1 se deduce que

$$CO = \boxed{m} = \frac{\overline{EC}}{\cos \frac{\alpha}{2}} = \frac{l_n : 2}{\sqrt{\frac{3 - 2 \cos \frac{\pi}{n}}{4}}} = \frac{1}{2 \sqrt{\frac{3 - 2 \cos \frac{\pi}{n}}{4}}} l_n =$$

$$l_n = \frac{1}{\sqrt{3 - 2 \cos \frac{\pi}{n}}} l_n = \sqrt{\frac{1}{3 - 2 \cos \frac{\pi}{n}}} l_n$$

* Cálculo logarítmico

$$\begin{aligned} \lg 1 &= 0, \\ - \lg 1,1980620 &= -0,0764591 \\ \lg 1,9215409 &= 0,2835409 \\ \frac{1}{2} \lg 1,9215409 &= 0,14177045 \\ \text{Antilog } 0,14177045 &= 1,380389 \end{aligned}$$

Para el caso del dibujo $n = 7$, sera: ↑

$$l_7 = \sqrt{\frac{1}{3 - 2 \cos (180^\circ : 7)}} l_7 = \sqrt{\frac{1}{3 - 2 \cos 25^\circ 42' 51,4''}} l_7 =$$

$$\sqrt{\frac{1}{3 - 2 \times 0,9009690}} \times l_7 = \sqrt{\frac{1}{1,1980620}} \times l_7 =$$

$$0,9136089... \times l_7 = 0,9136089... \times$$

$$\begin{aligned} \lg \cos 25^\circ 42' 51,4'' &= \\ &= 0,9547098 \end{aligned}$$

$$\begin{aligned} \text{Antilog } 0,9547098 &= \\ &= 0,9009690 \end{aligned}$$

Radio "a" de la esfera circunscrita

Se obtiene aplicando la fórmula general [1] (ver lámina 33)

$$a = \frac{l_n^2}{2 \sqrt{l_n^2 - m^2}} = \frac{l_n^2}{2 \sqrt{l_n^2 - \left(\sqrt{\frac{1}{3 - 2 \cos \frac{\pi}{n}}} \times l_n \right)^2}} = \frac{1}{2 \sqrt{1 - \frac{1}{3 - 2 \cos \frac{\pi}{n}}}} \times l_n =$$

$$= \frac{1}{2 \sqrt{\frac{3 - 2 \cos \frac{\pi}{n} - 1}{3 - 2 \cos \frac{\pi}{n}}}} l_n = \sqrt{\frac{3 - 2 \cos \frac{\pi}{n}}{4 (2 - 2 \cos \frac{\pi}{n})}} l_n = \sqrt{\frac{3 - 2 \cos \frac{\pi}{n}}{8 (1 - \cos \frac{\pi}{n})}} \times l_n$$

Para el caso del dibujo $n = 7$, sera:

$$a = \sqrt{\frac{3 - 2 \cos (180^\circ : 7)}{8 (1 - \cos (180^\circ : 7))}} \times l_n = \sqrt{\frac{3 - 2 \cos 25^\circ 41' 51,4''}{8 (1 - \cos 25^\circ 41' 51,4'')}} \times l_n =$$

$$= \sqrt{\frac{3 - 2 \times 0,9009690}{8 (1 - 0,9009690)}} \times l_n = \sqrt{\frac{1,1980620}{0,7922380}} \times l_n = 1,2297280 \times l_n$$

Let $f(x) = x^2 + 2x + 1$ and $g(x) = x^2 - 2x + 1$.
Then $f(x) + g(x) = (x^2 + 2x + 1) + (x^2 - 2x + 1) = 2x^2 + 2$.
Also $f(x) - g(x) = (x^2 + 2x + 1) - (x^2 - 2x + 1) = 4x$.
Hence $\frac{f(x) + g(x)}{f(x) - g(x)} = \frac{2x^2 + 2}{4x} = \frac{x^2 + 1}{2x}$.

Let $f(x) = x^2 + 2x + 1$ and $g(x) = x^2 - 2x + 1$.
Then $f(x) + g(x) = 2x^2 + 2$ and $f(x) - g(x) = 4x$.
Hence $\frac{f(x) + g(x)}{f(x) - g(x)} = \frac{x^2 + 1}{2x}$.

$$\frac{x^2 + 1}{2x} = \frac{x^2}{2x} + \frac{1}{2x} = \frac{x}{2} + \frac{1}{2x}$$

Let $f(x) = x^2 + 2x + 1$ and $g(x) = x^2 - 2x + 1$.
Then $f(x) + g(x) = 2x^2 + 2$ and $f(x) - g(x) = 4x$.
Hence $\frac{f(x) + g(x)}{f(x) - g(x)} = \frac{x^2 + 1}{2x}$.

Cálculo logarítmico anterior

$$\begin{aligned} \lg 1.198062 &= 0,0784793 \\ - \lg 0.792248 &= -7,8988611 \\ \hline &0,1796182 \end{aligned}$$

$$\frac{1}{2} \times 0.1796182 = 0,0898091$$

$$\text{Antilog } 0,0898091 = 1,2297280 \dots$$

Para el caso del dibujo $n = 7$ y $a = 55,0$ será:

$$a = 55 \text{ mm.} \quad \lg = \frac{55}{1,229728} = 44,725 \text{ mm.}$$

Radio "b" de la esfera tangente a las aristas

Se obtiene aplicando la fórmula general [3] (ver lám. 33)

$$\begin{aligned} b &= \sqrt{a^2 - \frac{l_n^2}{4}} = \sqrt{\left(\sqrt{\frac{3-2\cos\frac{\pi}{n}}{8(1-\cos\frac{\pi}{n})}} \times l_n\right)^2 - \frac{1}{4} l_n^2} = \\ &= \sqrt{\frac{3-2\cos\frac{\pi}{n}}{8(1-\cos\frac{\pi}{n})}} - \frac{1}{4} \times l_n = \sqrt{\frac{12-8\cos\frac{\pi}{n} - 2(1-\cos\frac{\pi}{n})}{32(1-\cos\frac{\pi}{n})}} \times l_n = \\ &= \sqrt{\frac{12-8\cos\frac{\pi}{n} - 2 + 2\cos\frac{\pi}{n}}{32(1-\cos\frac{\pi}{n})}} \times l_n = \sqrt{\frac{4}{32(1-\cos\frac{\pi}{n})}} \times l_n = \\ &= \sqrt{\frac{1}{8(1-\cos\frac{\pi}{n})}} \times l_n \end{aligned}$$

Para el caso del dibujo $n = 7$, será:

Handwritten text line.

Handwritten text block, possibly a list or set of instructions.

Handwritten text line.

Handwritten text line.

Handwritten text line.

Handwritten text line.

Handwritten text line.

Handwritten text line.

Handwritten text line.

Handwritten text line.

Handwritten text line.

Handwritten text line.

Handwritten text line, possibly a signature or date.

Handwritten text line at the bottom of the page.

$$b = \sqrt{\frac{1}{8(1 - \cos(180:7))}} \times l_7 = \sqrt{\frac{1}{8(1 - \cos 25^\circ 41' 51.4'')}} \times l_7 =$$

$$= \sqrt{\frac{1}{8(1 - 0.9009690)}} \times l_7 = \sqrt{\frac{1}{0.7922480}} \times l_7 = 1.123490... \times 44.725 =$$

$$= 50.2 \text{ mm.}$$

Radio "d₃" de la circunferencia circunscrita a una cara triangular

Se demuestra en Geometría es

$$d_3 = \frac{\sqrt{3}}{3} l_n = 0.57735027... l_n$$

Para el caso del dibujo, será: $d_3 = 0.57735027... \times 44.725 = 25.8 \text{ mm}$

Radio "d_n" de la circunferencia circunscrita a una cara regular de "n" lados.

En la página 5, fórmula [3], hemos determinado su valor.

$$d_n = \frac{l_n}{2} : \operatorname{sen} \frac{\pi}{n}$$

Para el caso del dibujo, $n = 7$, será:

$$d_7 = \frac{1}{2 \operatorname{sen} \frac{\pi}{n}} l_n = \frac{1}{2 \operatorname{sen}(180:7)} l_n = \frac{1}{2 \operatorname{sen}(25^\circ 41' 51.4'')} l_n =$$

$$= \frac{1}{2 \times 0.4338836} \times l_n = 1.1523828... \times 44.725 = 51.5 \text{ mm}$$

Q.1. Write the following in standard form.

$$1. \quad 1000000 + 100000 + 10000 + 1000 + 100 + 10 + 1$$

Sol.

$$1000000 + 100000 + 10000 + 1000 + 100 + 10 + 1$$

1111111

Q.2. Write the following in standard form.

$$2. \quad 1000000 + 100000 + 10000 + 1000 + 100 + 10 + 1$$

$$1000000 + 100000 + 10000 + 1000 + 100 + 10 + 1$$

1111111

$$3. \quad 1000000 + 100000 + 10000 + 1000 + 100 + 10 + 1$$

1111111

$$4. \quad 1000000 + 100000 + 10000 + 1000 + 100 + 10 + 1$$

$$1000000 + 100000 + 10000 + 1000 + 100 + 10 + 1$$

1111111

$$5. \quad 1000000 + 100000 + 10000 + 1000 + 100 + 10 + 1$$

$$1000000 + 100000 + 10000 + 1000 + 100 + 10 + 1$$

$$1000000 + 100000 + 10000 + 1000 + 100 + 10 + 1$$

Radio " c_3 " de la esfera tangente a las caras triangulares de lado " l_n "

Se obtiene aplicando la fórmula general [2] (ver lám. 33)

$$c_3 = \sqrt{a^2 - (d_3)^2} = \sqrt{\left(\sqrt{\frac{3-2\cos(\pi:n)}{8(1-\cos(\pi:n))}} \times l_n\right)^2 - \left(\frac{\sqrt{3}}{3} l_n\right)^2} =$$

$$= \sqrt{\frac{3-2\cos(\pi:n)}{8(1-\cos(\pi:n))}} - \frac{1}{3} \times l_n$$

Para el caso del dibujo $n=7$, será: (ver cálculo numérico "a")

$$c_3 = \sqrt{\frac{3-2\cos(180^\circ:7)}{8(1-\cos(180^\circ:7))}} - \frac{1}{3} \times l_n = \sqrt{\frac{1,1980620}{0,7922480}} - \frac{1}{3} \times l_n =$$

$$= \sqrt{\frac{3 \times 1,1980620 - 0,792248}{3 \times 0,792248}} \times l_n = \sqrt{\frac{2,801938}{2,376744}} \times l_n =$$

$$= 1,0857588... \times 44,725 = 48,6 \text{ mm}$$

Radio " c_n " de la esfera tangente a las caras regulares de " n " lados.

Aplicando la fórmula general [2] (ver lám. 33)

$$c_n = \sqrt{a^2 - (d_n)^2} = \sqrt{\left(\sqrt{\frac{3-2\cos(\pi:n)}{8(1-\cos(\pi:n))}} \times l_n\right)^2 - \left(\frac{l_n}{2\sin(\pi:n)}\right)^2} =$$

$$= \sqrt{\frac{3-2\cos(\pi:n)}{8(1-\cos(\pi:n))}} - \left(\frac{1}{2\sin(\pi:n)}\right) \times l_n$$



Para el caso del dibujo, será $n = 7$ (ver cál. num. de "a" y "d₇")

$$C_7 = \sqrt{\frac{3 - 2 \cos(\pi:7)}{8(1 - \cos(\pi:7))}} - \frac{1}{[2 \operatorname{sen}(\pi:7)]^2} \times l_7 = \sqrt{\frac{1.1980620}{0.7922480}} - \left(\frac{1}{0.8677672}\right)^2$$

$$= \sqrt{1.5122310} - 1.3279861 \times l_n = \sqrt{0.1842449} \times l_n = 0.4292376 \times l_n =$$

$$= 0.4292376 \dots \times 44.735 = 19.2 \text{ mm.}$$

Ángulo rectilíneo " α_3 " del diedro formado por una cara triangular, con el plano diametral del arquimedianos que pasa por una arista de aquélla

Se obtiene, en función de su tangente, por la fórmula general [5] (ver lám. 33)

$$\boxed{tg \alpha_3} = \frac{2 C_3}{\sqrt{4(d_3)^2 - l_n^2}} = \frac{2 C_3}{\sqrt{4 \times \left(\frac{\sqrt{3}}{3}\right)^2 - 1}} \times l_n = \frac{2 C_3}{\sqrt{\frac{1}{3}} l_n} = 2\sqrt{3} \times \frac{C_3}{l_n} =$$

$$= 2\sqrt{3} \sqrt{\frac{3 - 2 \cos(\pi:n)}{8(1 - \cos(\pi:n))}} - \frac{1}{3} \times \frac{l_n}{l_n} = \boxed{2 \sqrt{\frac{9 - 6 \cos(\pi:n)}{8(1 - \cos(\pi:n))}} - 1}$$

Para el caso del dibujo, $n = 7$; será

$$tg \alpha_3 = 2\sqrt{3} \times \sqrt{\frac{3 - 2 \cos(\pi:7)}{8(1 - \cos(\pi:7))}} - \frac{1}{3} = 2\sqrt{3} \times 1.0857688 \dots =$$

$$= 3.4641016 \dots \times 1.0857688 \dots = 3.7612134 \dots$$

Received of the Treasurer of the University of Chicago

the sum of \$100.00

for the purchase of books

for the Library of the University of Chicago

for the year 1910

for the purchase of books

for the year 1910

for the purchase of books

for the year 1910

for the purchase of books

for the year 1910

$$\tan \alpha_3 = \tan 3,7612134 = 0,5753280$$

$$\alpha_3 = 75^\circ 6' 40,0''$$

Ángulo rectilíneo " α_n " del diedro formado por una cara regular de " n " lados, con el plano diametral del arquimédiano que pasa por una arista de aquella.

Se obtiene, en función de su tangente, por la fórmula general [5] (ver lám. 33)

$$\begin{aligned} \boxed{\tan \alpha_n} &= \frac{2 C_n}{\sqrt{4 (d_n)^2 - l^2}} = \frac{2 C_n}{\sqrt{4 \left(\frac{l_n}{2 \operatorname{sen}(\pi:n)} \right)^2 - l_n^2}} = \frac{2 C_n}{\sqrt{\frac{1}{\operatorname{sen}^2(\pi:n)} - 1} \times l_n} \\ &= 2 \sqrt{\frac{3-2 \cos(\pi:n)}{8(1-\cos(\pi:n))}} - \left(\frac{1}{2 \operatorname{sen}(\pi:n)} \right)^2 \times l_n : \sqrt{\frac{1-\operatorname{sen}^2(\pi:n)}{-\operatorname{sen}^2(\pi:n)}} \times l_n = \\ &= 2 \sqrt{\frac{3-2 \cos(\pi:n)}{8(1-\cos(\pi:n))}} - \left(\frac{1}{2 \operatorname{sen}(\pi:n)} \right)^2 : \sqrt{\frac{\cos^2(\pi:n)}{\operatorname{sen}^2(\pi:n)}} = \\ &= 2 \sqrt{\frac{3-2 \cos(\pi:n)}{8(1-\cos(\pi:n))}} - \left(\frac{1}{2 \operatorname{sen}(\pi:n)} \right)^2 : \frac{1}{\tan^2(\pi:n)} = \\ &= 2 \tan^2(\pi:n) \sqrt{\frac{3-2 \cos(\pi:n)}{8(1-\cos(\pi:n))}} - \left(\frac{1}{2 \operatorname{sen}(\pi:n)} \right)^2 \end{aligned}$$

Para el caso del dibujo $n = 7$, será

$$\tan \alpha_7 = 2 \tan^2(\pi:7) \times \sqrt{\frac{3-2 \cos(\pi:7)}{8(1-\cos(\pi:7))}} - \left(\frac{1}{2 \operatorname{sen}(\pi:7)} \right)^2 =$$

1. The first part of the book is devoted to a general survey of the subject.

2. The second part is devoted to a detailed study of the various aspects of the subject.

3. The third part is devoted to a study of the various aspects of the subject, and the fourth part is devoted to a study of the various aspects of the subject.

4. The fifth part is devoted to a study of the various aspects of the subject, and the sixth part is devoted to a study of the various aspects of the subject.

5. The seventh part is devoted to a study of the various aspects of the subject, and the eighth part is devoted to a study of the various aspects of the subject.

6. The ninth part is devoted to a study of the various aspects of the subject, and the tenth part is devoted to a study of the various aspects of the subject.

7. The eleventh part is devoted to a study of the various aspects of the subject, and the twelfth part is devoted to a study of the various aspects of the subject.

8. The thirteenth part is devoted to a study of the various aspects of the subject, and the fourteenth part is devoted to a study of the various aspects of the subject.

9. The fifteenth part is devoted to a study of the various aspects of the subject, and the sixteenth part is devoted to a study of the various aspects of the subject.

10. The seventeenth part is devoted to a study of the various aspects of the subject, and the eighteenth part is devoted to a study of the various aspects of the subject.

$$= 2 \operatorname{tg} (25^{\circ} 41' 51,4'') \times 0,42 \ 92 \ 37 \ 6 \dots$$

Cálculo logarítmico

$$\begin{array}{rcl} \lg \operatorname{tg} \alpha_7 & = & \lg 2 \quad \quad \quad = 0,30 \ 10 \ 30 \ 0 \\ + & & \lg \operatorname{tg} (25^{\circ} 41' 51,4'') = 7,68 \ 23 \ 40 \ 1 \ + \\ + & & \lg 0,42 \ 92 \ 37 \ 6 = 7,63 \ 26 \ 97 \ 8 \ + \\ & & \underline{\lg \operatorname{tg} \alpha_7} = \underline{7,61 \ 60 \ 67 \ 9} \end{array}$$

$$\alpha_7 = 22^{\circ} \ 26' \ 46,0''$$

Ángulo rectilíneo " φ_{3-n} " del diedro formado por una cara triangular y otra regular de " n " lados

Aplicando la fórmula general [4] (ver lám. 33)

$$\boxed{\varphi_{3-n} = \alpha_3 + \alpha_n}$$

Para el caso del dibujo, $n = 7$, será

$$\begin{aligned} \varphi_{3-7} &= \alpha_3 + \alpha_7 = 75^{\circ} \ 6' \ 40,0'' + 22^{\circ} \ 26' \ 46,0'' = \\ &= 97^{\circ} \ 33' \ 26,0'' \end{aligned}$$

Ángulo rectilíneo " φ_{3-3} " del diedro formado por dos caras triangulares.

Aplicando la fórmula general [4] (ver lám. 33)

$$\boxed{\varphi_{3-3}} = \alpha_3 + \alpha_2 = \boxed{2\alpha_3}$$

Para el caso del dibujo, será: ($n = 7$)

$$\varphi_{3-3} = 2 \cdot (75^\circ 6' 40,0'') = 150^\circ 13' 20,0''$$

Área lateral "S" del arquimedianos

Se compone de la suma de " $2n$ " caras triangulares y de (para cualquier valor natural de $n > 3$) siempre 2 caras regulares de " n " lados.

La apotema de una cara triangular es de $\frac{\sqrt{3}}{6} l_n$; el área de una cara será pues $\frac{3l}{2} \times \frac{\sqrt{3}}{6} l_n = \frac{\sqrt{3}}{4} (l_n)^2$.

La apotema de una cara regular de " n " lados, la hemos obtenido en [2] (ver foja n° 4), y es

$$k_n = \frac{l_n}{2 \tan(\pi/n)}$$

El área total será pues:

$$\boxed{S} = 2n \frac{\sqrt{3}}{4} (l_n)^2 + 2 \left(\frac{n \times l_n}{2} \times \frac{l_n}{2 \tan(\pi/n)} \right) = \frac{\sqrt{3}}{2} n (l_n)^2 +$$

$$+ \frac{1}{2 \tan(\pi/n)} n (l_n)^2 = \left(\frac{\sqrt{3}}{2} + \frac{\cot(\pi/n)}{2} \right) n (l_n)^2 = \boxed{\frac{\sqrt{3} + \cot(\pi/n)}{2} n (l_n)^2}$$

Volumen "V" del arquimedianos

Se compone de la suma de " $2n$ " pirámides re-

THE NEW YORK PUBLIC LIBRARY

ASTOR LENOX TILDEN FOUNDATION
 410 Fifth Avenue, New York
 This book is loaned to you by the
 New York Public Library
 and is to be returned to the
 Library when called for.
 The book is to be kept in good
 condition and is not to be
 sold, lent, or otherwise disposed
 of without the permission of the
 Library.
 The book is to be kept in good
 condition and is not to be
 sold, lent, or otherwise disposed
 of without the permission of the
 Library.

THE NEW YORK PUBLIC LIBRARY
 ASTOR LENOX TILDEN FOUNDATION
 410 Fifth Avenue, New York
 This book is loaned to you by the
 New York Public Library
 and is to be returned to the
 Library when called for.
 The book is to be kept in good
 condition and is not to be
 sold, lent, or otherwise disposed
 of without the permission of the
 Library.

gular de base triangular y altura " c_3 ", y de 2 pirámides de base regular de " n " lados y altura " c_n ".
Su valor será pues:

$$\begin{aligned}
 V &= 2n \left(\frac{\sqrt{3}}{4} (l_n)^2 \right) \times \frac{c_3}{3} + 2 \left(\frac{n l_n}{2} \times \frac{l_n}{2 \tan(\pi/n)} \right) \times \frac{c_n}{3} = \\
 &= \frac{n \sqrt{3}}{6} (l_n)^2 \times \frac{\sqrt{3-2 \cos(\pi/n)} - 1}{8(1-\cos(\pi/n))} + \frac{1}{3} \times l_n + \frac{n (l_n)^2}{6 \tan(\pi/n)} \times \\
 &\times \frac{\sqrt{3-2 \cos(\pi/n)} - \left(\frac{1}{2 \tan(\pi/n)} \right)^2}{8(1-\cos(\pi/n))} \times l_n = \\
 &= \left(\frac{1}{6} \times \frac{\sqrt{9-6 \cos(\pi/n)} - 1}{8(1-\cos(\pi/n))} + \frac{\cotg(\pi/n)}{6} \times \frac{\sqrt{3-2 \cos(\pi/n)} - \left(\frac{1}{2 \tan(\pi/n)} \right)^2}{8(1-\cos(\pi/n))} \right) n (l_n)^3
 \end{aligned}$$

FIGURA CORPÓREA

Se obtiene por acoplamiento de " $2n$ " triángulos equiláteros de lado " l_n ", y de 2 caras regulares de " n " lados de igual magnitud " l_n ". El acoplamiento deberá hacerse de forma que en cada vértice concurren 3 triángulos y un polígono regular de " n " lados.

TO THE HONORABLE THE PRESIDENT OF THE UNIVERSITY OF CHICAGO
FROM THE PHYSICS DEPARTMENT

RECEIVED JANUARY 10 1927

THE UNIVERSITY OF CHICAGO

CHICAGO, ILLINOIS

TO THE HONORABLE THE PRESIDENT OF THE UNIVERSITY OF CHICAGO

FROM THE PHYSICS DEPARTMENT

RECEIVED JANUARY 10 1927
THE UNIVERSITY OF CHICAGO
CHICAGO, ILLINOIS

En el cuadro sinóptico que damos a continuación, se resumen los resultados analíticos obtenidos anteriormente.

CUADRO SINÓPTICO

Magnitud	Valor exacto	Valor decimal aproximado
a	$\sqrt{\frac{3-2\cos(\pi:n)}{8(1-\cos(\pi:n))}} \times l_n$	Variable con $n > 3$
b	$\sqrt{\frac{1}{8(1-\cos(\pi:n))}} \times l_n$	Variable con $n > 3$
c_3	$\sqrt{\frac{3-2\cos(\pi:n)}{8(1-\cos(\pi:n))}} - \frac{1}{3} \times l_n$	Variable con $n > 3$
c_n	$\sqrt{\frac{3-2\cos(\pi:n)}{8(1-\cos(\pi:n))}} - \left(\frac{1}{2\sin(\pi:n)}\right)^2 \times l_n$	Variable con $n > 3$
d_3	$\frac{\sqrt{3}}{3} l_n$	0.57 73 50... l_n
d_n	$\frac{1}{2\sin(\pi:n)} \cdot l_n$	Variable con $n > 3$
m	$\sqrt{\frac{1}{3-2\cos(\pi:n)}} \times l_n$	Variable con $n > 3$
α_3	$tg \alpha_3 = 2 \times \sqrt{\frac{9-6\cos(\pi:n)}{8(1-\cos(\pi:n))}} - 1$	Variable con $n > 3$
α_n	$tg \alpha_n = 2 \times tg(\pi:n) \sqrt{\frac{3-2\cos(\pi:n)}{8(1-\cos(\pi:n))}} - \left(\frac{1}{2\sin(\pi:n)}\right)^2$	Variable con $n > 3$
φ_{3-n}	$\varphi_{3-n} = \alpha_3 + \alpha_n$	Variable con $n > 3$
φ_{3-3}	$\varphi_{3-3} = 2 \alpha_3$	Variable con $n > 3$
S	$\frac{\sqrt{3} + ctg(\pi:n)}{2} \times n(l_n)^2$	Variable con $n > 3$
V	$V = \left[\frac{1}{6} \times \sqrt{\frac{9-6\cos(\pi:n)}{8(1-\cos(\pi:n))}} - 1 + \frac{ctg(\pi:n)}{6} \times \sqrt{\frac{3-2\cos(\pi:n)}{8(1-\cos(\pi:n))}} - \left(\frac{1}{2\sin(\pi:n)}\right)^2\right] \times n(l_n)^3$	Variable con $n > 3$

The following table shows the results of the experiments conducted on the 10th of the month.

Table showing the results of the experiments.

Time	Temperature	Pressure	Volume	Weight
1.00	20.0	1.00	1.00	1.00
2.00	20.0	1.00	1.00	1.00
3.00	20.0	1.00	1.00	1.00
4.00	20.0	1.00	1.00	1.00
5.00	20.0	1.00	1.00	1.00
6.00	20.0	1.00	1.00	1.00
7.00	20.0	1.00	1.00	1.00
8.00	20.0	1.00	1.00	1.00
9.00	20.0	1.00	1.00	1.00
10.00	20.0	1.00	1.00	1.00
11.00	20.0	1.00	1.00	1.00
12.00	20.0	1.00	1.00	1.00
13.00	20.0	1.00	1.00	1.00
14.00	20.0	1.00	1.00	1.00
15.00	20.0	1.00	1.00	1.00
16.00	20.0	1.00	1.00	1.00
17.00	20.0	1.00	1.00	1.00
18.00	20.0	1.00	1.00	1.00
19.00	20.0	1.00	1.00	1.00
20.00	20.0	1.00	1.00	1.00
21.00	20.0	1.00	1.00	1.00
22.00	20.0	1.00	1.00	1.00
23.00	20.0	1.00	1.00	1.00
24.00	20.0	1.00	1.00	1.00
25.00	20.0	1.00	1.00	1.00
26.00	20.0	1.00	1.00	1.00
27.00	20.0	1.00	1.00	1.00
28.00	20.0	1.00	1.00	1.00
29.00	20.0	1.00	1.00	1.00
30.00	20.0	1.00	1.00	1.00
31.00	20.0	1.00	1.00	1.00
32.00	20.0	1.00	1.00	1.00
33.00	20.0	1.00	1.00	1.00
34.00	20.0	1.00	1.00	1.00
35.00	20.0	1.00	1.00	1.00
36.00	20.0	1.00	1.00	1.00
37.00	20.0	1.00	1.00	1.00
38.00	20.0	1.00	1.00	1.00
39.00	20.0	1.00	1.00	1.00
40.00	20.0	1.00	1.00	1.00
41.00	20.0	1.00	1.00	1.00
42.00	20.0	1.00	1.00	1.00
43.00	20.0	1.00	1.00	1.00
44.00	20.0	1.00	1.00	1.00
45.00	20.0	1.00	1.00	1.00
46.00	20.0	1.00	1.00	1.00
47.00	20.0	1.00	1.00	1.00
48.00	20.0	1.00	1.00	1.00
49.00	20.0	1.00	1.00	1.00
50.00	20.0	1.00	1.00	1.00
51.00	20.0	1.00	1.00	1.00
52.00	20.0	1.00	1.00	1.00
53.00	20.0	1.00	1.00	1.00
54.00	20.0	1.00	1.00	1.00
55.00	20.0	1.00	1.00	1.00
56.00	20.0	1.00	1.00	1.00
57.00	20.0	1.00	1.00	1.00
58.00	20.0	1.00	1.00	1.00
59.00	20.0	1.00	1.00	1.00
60.00	20.0	1.00	1.00	1.00
61.00	20.0	1.00	1.00	1.00
62.00	20.0	1.00	1.00	1.00
63.00	20.0	1.00	1.00	1.00
64.00	20.0	1.00	1.00	1.00
65.00	20.0	1.00	1.00	1.00
66.00	20.0	1.00	1.00	1.00
67.00	20.0	1.00	1.00	1.00
68.00	20.0	1.00	1.00	1.00
69.00	20.0	1.00	1.00	1.00
70.00	20.0	1.00	1.00	1.00
71.00	20.0	1.00	1.00	1.00
72.00	20.0	1.00	1.00	1.00
73.00	20.0	1.00	1.00	1.00
74.00	20.0	1.00	1.00	1.00
75.00	20.0	1.00	1.00	1.00
76.00	20.0	1.00	1.00	1.00
77.00	20.0	1.00	1.00	1.00
78.00	20.0	1.00	1.00	1.00
79.00	20.0	1.00	1.00	1.00
80.00	20.0	1.00	1.00	1.00
81.00	20.0	1.00	1.00	1.00
82.00	20.0	1.00	1.00	1.00
83.00	20.0	1.00	1.00	1.00
84.00	20.0	1.00	1.00	1.00
85.00	20.0	1.00	1.00	1.00
86.00	20.0	1.00	1.00	1.00
87.00	20.0	1.00	1.00	1.00
88.00	20.0	1.00	1.00	1.00
89.00	20.0	1.00	1.00	1.00
90.00	20.0	1.00	1.00	1.00
91.00	20.0	1.00	1.00	1.00
92.00	20.0	1.00	1.00	1.00
93.00	20.0	1.00	1.00	1.00
94.00	20.0	1.00	1.00	1.00
95.00	20.0	1.00	1.00	1.00
96.00	20.0	1.00	1.00	1.00
97.00	20.0	1.00	1.00	1.00
98.00	20.0	1.00	1.00	1.00
99.00	20.0	1.00	1.00	1.00
100.00	20.0	1.00	1.00	1.00

PROCESO GRÁFICO-ANALÍTICO

Después del cálculo de las magnitudes principales, vamos a proceder en la lámina 46, a la representación gráfica de un Arquimedianos de la Serie A_n , en el caso particular de ser $n = 7$.

Para su trazado nos valdremos de las cotas calculadas por las fórmulas anteriores, determinadas previamente para el caso particular que nos ocupa. Dichas magnitudes las obtendremos en función del lado " l_7 " del arquimedianos, cuya longitud es de 44,725 mm.

El cálculo de dichas magnitudes se efectúa a continuación:

$$l_7 = \text{Dato del ejercicio} = 44,7 \text{ mm}$$

$$a = 1,22 \ 77 \ 28 \ 0 \dots \times 44,725 = 55,0 \text{ mm}$$

$$b = 1,12 \ 34 \ 70 \ 0 \dots \times 44,725 = 50,2 \text{ mm}$$

$$C_3 = 1,08 \ 57 \ 68 \ 8 \times 44,725 = 48,6 \text{ mm}$$

$$C_7 = 0,42 \ 92 \ 37 \ 6 \times 44,725 = 19,2 \text{ mm}$$

$$d_3 = 0,57 \ 73 \ 50 \ 3 \dots \times 44,725 = 25,8 \text{ mm}$$

$$d_7 = 1,15 \ 23 \ 82 \ 2 \dots \times 44,725 = 51,5 \text{ mm}$$

Para facilitar el trazado gráfico, hemos elegido la posición del arquimedianos A_7 (7 en general A_n) de forma que sus caras regulares de 7 lados (siempre son do), sean paralelas a II, y uno de sus lados perpendicular a I; de esta forma obtenemos en I la verdadera amplitud

del diedro que forma esa cara, con la triangular contigua cuya arista común es dicho lado de " P_n " perpendicular a I.

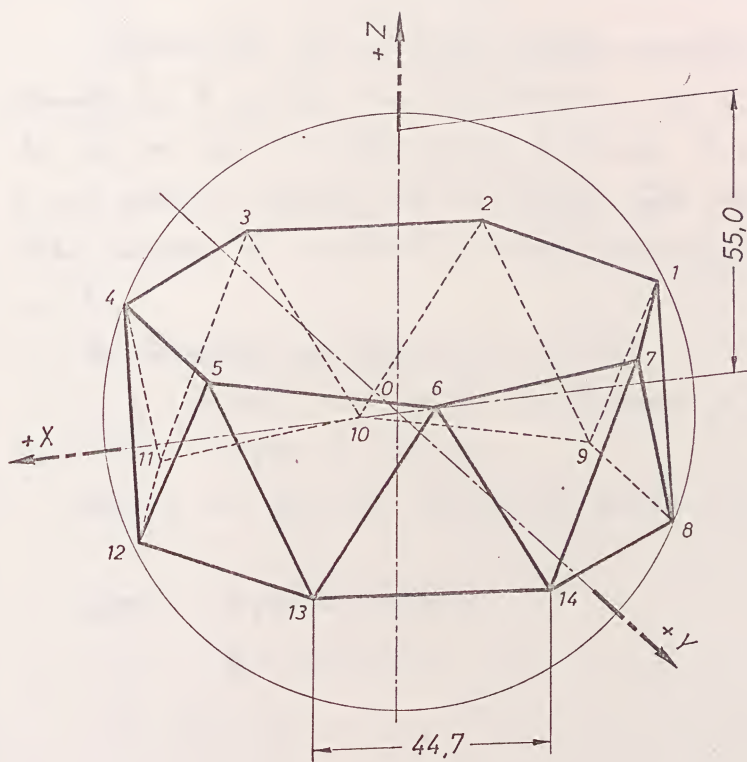
En estas condiciones, la proyección ^(sobre II) de la cara superior P_7 será un polígono regular de 7 lados y radio " d_7 ", con uno de sus lados perpendicular a I y el de la otra cara inferior P_7 , otro polígono igual, concéntricos con el anterior y girado con respecto a éste un ángulo " $\pi:14$ " (en general " $\pi:2n$ "); el contorno aparente de esta proyección sobre II será un polígono regular de 14 lados (en general de " $2n$ " lados) inscrito en la misma circunferencia de radio " d_7 " conocido.

Teniendo presente lo expuesto anteriormente, el orden de operaciones del trazado gráfico (lámina 46), es el siguiente:

- 1° Situar el centro O de coordenadas $O(72, 72, 85)$ mm.
- 2° Dibujar en I, II y III las proyecciones de la esfera circunscrita de 55,0 mm de radio
- 3° Dibujar la proyección sobre II del Arquimedianos, trazando con centro O_{II} y radio " d_7 " una circunferencia que se dividirá en 14 partes iguales (en general " $2n$ ") tomando como origen de división el radio OT_1 , paralelo a $+X$. Después de efectuada la división unir éstas en la forma que se indica en la lámina con lo que obteniéndose la proyección buscada (comprobar

las longitudes de " l_7 " y " b ")

2° Conocida la proyección sobre II del arquimedianos, y basándose en ella, puede obtenerse seguidamente las proyecciones I y III, bastando para ello trazar dos paralelas al eje $+x$, equidistantes de O_I y O_{III} , y con la separación " c_7 ". Sobre la superior, estarán situados los vértices 1 al 7, y sobre la inferior los 8 al 14; las proyecciones de dichos vértices habrán de corresponderse con las ya obtenidas en el plano II. (comprobar la longitud de " c_3 " y la amplitud de " φ_{3-7} ").



$$n = 7$$

Arquimedianos Serie A_n



Fig. 1

Geometrische Optik

ENUNCIADO

Representar, por el método gráfico-analítico, en los planos I, II y III, un Arquimédiano de la Serie B_n , en el que en cada vértice concurren 2 cuadrados y un polígono regular de " n " lados, todos de igual lado, siendo " n " natural y mayor que 2, excepto $n = 4$.

La longitud del lado, para $n = 9$,

es de 35.6 mm, y las coordenadas de su centro O , son: $O (72, 72, 85)$ mm.

Dibujar en formato A3V y a escala 1:1

DATOS: $O (72, 72, 85)$ mm

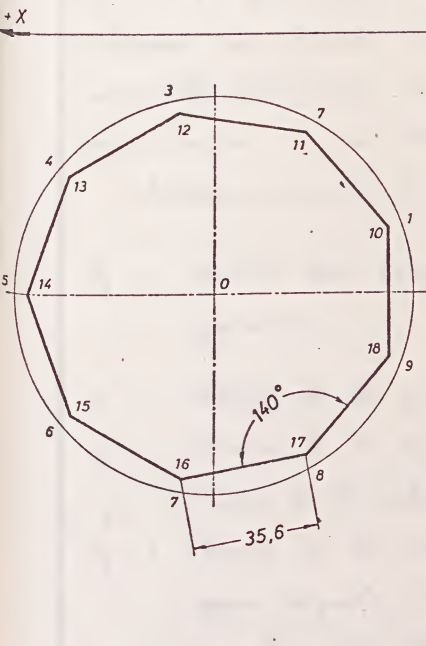
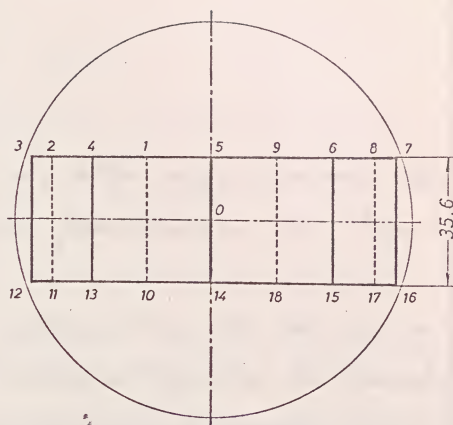
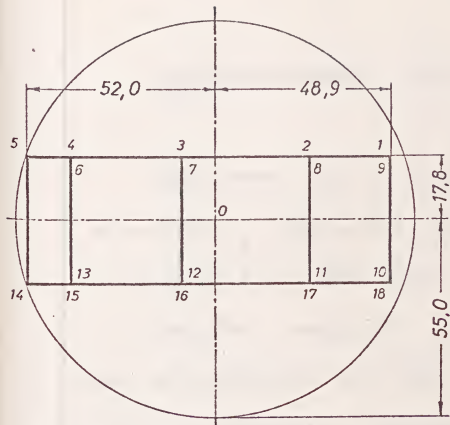
$$l_{B_9} = 35,6 \text{ mm}$$

Dear Sir,

I am writing to you to inform you that I have received your letter of the 10th inst. and in reply to inform you that the same has been forwarded to the proper authorities for their consideration. I am sorry that I cannot give you a more definite answer at this time, but I am sure that you will understand the necessity of this course.

I am, Sir, very respectfully,
Your obedient servant,
J. H. [Name]

[Signature]
[Address]



ARQUIMEDIANOS Serie B_n

Número de caras cuadradas.....	$C_4 = n$
Número de caras regulares de "n" lados.....	$C_n = 2$
Número de vértices.....	$V = 2n$
Número de aristas.....	$A = 3n$
Número de caras de un ángulo sólido...	$2P_4 + 1P_n$

ENUNCIADO

Representar por el método gráfico-analítico, en los planos I, II y III, un Arquimediano de la Serie B_n, en el que en cada vértice concurren dos cuadrados y un polígono regular de "n" lados todos de igual longitud, siendo "n" un número natural igual a 3, o mayor que 4. La longitud del lado, para n=9, es de 35,6 mm, y las coordenadas de su centro son: O (72, 72, 85) mm.

Dibujar en formato A3v y a escala 1:1.

Propuesta	De entrega	Entregada	Calificación	(firma)	Escuela
Fecha:					Curso
Alumno:					
Escala	1:1				Arquimedianos Serie B _n
					Lámina 47
					Curso 19 - 15

CONSIDERACIONES PREVIAS

Seguiremos en el estudio de estos arquimedianos, las directrices y fórmulas generales planteadas en el "Arquimédiano I", lámina 33.

Existen infinitos arquimedianos de esta Serie B_n, ya que el número de lados del polígono regular P_n, puede ser un número natural mayor que 2, excepto n=4. El estudio que realizamos a continuación considera este polígono en general, para cualquier valor de "n", así como la longitud de su lado correspondiente que designaremos a su vez, en forma general "l_n".

Determinaremos las siguientes magnitudes:

l_n = Arista del Arquimédiano Serie B_n (dato del ejercicio).

a = Radio de la esfera circunscrita.

b = Radio de la esfera tangente a las aristas.

c_4 = Radio de la esfera tangente a las caras cuadradas

c_n = Radio de la esfera tangente a las caras del polígono regular de "n" lados que en todos los casos son tan sólo dos P_n.

d_4 = Radio de la circunferencia circunscrita a una cara cuadrada

- d_n = Radio de la circunferencia circunscrita a una cara regular de " n " lados.
- m = Radio de la circunferencia circunscrita al polígono obtenido al unir los extremos de las aristas de un ángulo sólido.
- α_n = Ángulo rectilíneo del diedro formado por una cara cuadrada, con el plano diametral del arquimédiano que pasa por una arista de aquella.
- α_n = Ángulo rectilíneo del diedro formado por una cara regular de " n " lados, con el plano diametral del arquimédiano que pasa por una arista de aquella.
- φ_{4-n} = Ángulo rectilíneo del diedro formado por una cara cuadrada y otra regular de " n " lados.
- φ_{n-4} = Ángulo rectilíneo del diedro formado por dos caras cuadradas.
- S = Superficie
- V = Volumen

Antes de proceder al cálculo de las magnitudes anteriores, observemos que todos los arquimedianos de esta serie B_n , son prismas rectos regulares de bases P_n y caras laterales cuadradas. Bajo este enfoque, los cálculos anteriores permiten una notable simplificación. No obstante, los desarrollaremos siguiendo el proceso

general establecido en el estudio del arquimediano I, lámina 33.

PROCESO GRÁFICO-ANALÍTICO

El estudio realizado de este arquimediano, nos indica que se compone de " n " caras cuadradas, 2 caras regulares de " n " lados; " $2n$ " vértices y " $3n$ " aristas.

En cada vértice concurren 2 caras cuadradas y una regular de " n " lados, todas de igual lado.

Así pues, tendremos que:

ARQUIMEDIANO " B_n " ($2P_4 + 1P_n$); $C_4 = n$; $C_n = 2$; $V = 2n$; $A = 3n$

Cálculo de sus magnitudes

Arista " l_n " del arquimediano

Dato del ejercicio

Radio " m " de la circunferencia circunscrita al polígono obtenido al unir los extremos de las tres aristas de un ángulo sólido.

Este polígono es un triángulo isósceles ABC (fig. 1)

THE UNIVERSITY OF CHICAGO PRESS
CHICAGO, ILL. 60607

THE UNIVERSITY OF CHICAGO PRESS

THE UNIVERSITY OF CHICAGO PRESS
CHICAGO, ILL. 60607

THE UNIVERSITY OF CHICAGO PRESS
CHICAGO, ILL. 60607

THE UNIVERSITY OF CHICAGO PRESS

THE UNIVERSITY OF CHICAGO PRESS
CHICAGO, ILL. 60607

THE UNIVERSITY OF CHICAGO PRESS
CHICAGO, ILL. 60607

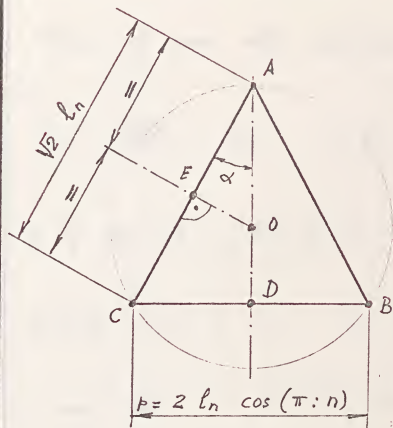


Figura 1

cuya base BC es la diagonal "p" del polígono regular C_n que une los extremos de dos lados consecutivos ($n > 2$, excepto $n=4$). El valor de \overline{CB} ha sido obtenido en la lám. 46, h 4, fig. 2, y es $\overline{CB} = p = 2 l_n \cos(\pi:n)$.

Los otros dos lados iguales \overline{AC} y \overline{AB} son diagonales de las caras cuadradas

que concurren en el ángulo sólido; su magnitud será pues $\overline{AC} = \overline{AB} = \sqrt{2} l_n$

De la figura se deduce:

$$\begin{aligned} \overline{AD} &= \sqrt{\overline{AC}^2 - \overline{CD}^2} = \sqrt{(\sqrt{2} l_n)^2 - \left(\frac{1}{2} p \cos(\pi:n) l_n\right)^2} = \\ &= \sqrt{2 - \cos^2(\pi:n)} \times l_n = \sqrt{1 + (1 - \cos^2(\pi:n))} l_n = \sqrt{1 + \sin^2(\pi:n)} l_n \end{aligned}$$

por lo que será:

$$\cos \alpha = \frac{\overline{AD}}{\overline{AC}} = \frac{\sqrt{1 + \sin^2(\pi:n)} \times l_n}{\sqrt{2} l_n} = \sqrt{\frac{1 + \sin^2(\pi:n)}{2}}$$

y en consecuencia:

$$AO = \boxed{m'} = \frac{\overline{AE}}{\cos \alpha} = \frac{\sqrt{2}}{2} l_n : \sqrt{\frac{1 + \sin^2(\pi:n)}{2}} = \sqrt{\left(\frac{\sqrt{2}}{2}\right)^2 : \frac{1 + \sin^2(\pi:n)}{2}} \times l_n =$$

$$= \sqrt{\frac{1}{1 + \sin^2(\pi:n)}} \times l_n$$

Para el caso del dibujo, $n = 9$, será:

$$\frac{\pi}{9} = \frac{180^\circ}{9} = 20^\circ \quad \lg \operatorname{sen} 20^\circ = \bar{1}, 53 \ 40 \ 517$$

$$2 \lg \operatorname{sen} 20^\circ = 2 \times \bar{1}, 53 \ 40 \ 517 = \bar{1}, 06 \ 81 \ 034$$

$$\operatorname{Antilog} \bar{1}, 06 \ 81 \ 034 = \operatorname{sen}^2 20^\circ = 0, 11 \ 69 \ 77 \ 8$$

$$1 + \operatorname{sen}^2 20^\circ = 1 + 0, 11 \ 69 \ 77 \ 8 = 1, 11 \ 69 \ 77 \ 8$$

$$\sqrt{\frac{1}{1 + \operatorname{sen}^2 20^\circ}} = \sqrt{\frac{1}{1, 11 \ 69 \ 77 \ 8}} = 0, 94 \ 61 \ 88 \ 5$$

$$\lg 1 = 0, 00 \ 00 \ 00 \ 0$$

$$- \lg 1, 11 \ 69 \ 77 \ 8 = -0, 04 \ 80 \ 44 \ 6$$

$$\bar{1}, 95 \ 19 \ 55 \ 4 : 2 = \bar{1}, 97 \ 59 \ 77 \ 7$$

$$\operatorname{Antilog} \bar{1}, 97 \ 59 \ 77 \ 7 = \underline{0, 94 \ 61 \ 88 \ 5 \dots}$$

$$m = 0, 94 \ 61 \ 88 \ 5 \dots \times$$

Radio "a" de la esfera circunscrita

Aplicando la fórmula general [1] (ver lám. 33)

$$a = \frac{l_n^2}{2 \sqrt{l_n^2 - m^2}} = \frac{l_n^2}{2 \sqrt{l_n^2 - \left(\sqrt{\frac{1}{1 + \operatorname{sen}^2 (\pi:n)}} \times l_n \right)^2}} =$$

$$= \frac{1}{2 \sqrt{1 - \frac{1}{1 + \operatorname{sen}^2 (\pi:n)}}} \times l_n = \frac{1}{2 \sqrt{\frac{\operatorname{sen}^2 (\pi:n)}{1 + \operatorname{sen}^2 (\pi:n)}}} \times l_n = \boxed{\frac{1}{2} \sqrt{\frac{1 + \operatorname{sen}^2 (\pi:n)}{\operatorname{sen}^2 (\pi:n)}} \times l_n}$$

Para el caso del dibujo, $n = 9$ será: (ver cálculo "m" hoja anterior)

$$\frac{\pi}{n} = \frac{180^\circ}{9} = 20^\circ$$

$$1 + \operatorname{sen}^2 20^\circ = 1,1169778; \quad \operatorname{sen}^2 20^\circ = 0,1169778..$$

$$a = \frac{1}{2} \sqrt{\frac{1,1169778}{0,1169778}}$$

$$\lg 1,1169778 = 0,0480446$$

$$- \lg 0,1169778 = -\bar{7},0681034$$

$$\frac{1}{2} \times 0,9799412 = 0,4899706$$

$$0,9799412$$

$$- \lg 2 = -0,3010300$$

$$\underline{a} = \text{antilog } 0,1889406 \times \lg = \underline{1,5450431} \times \lg$$

$$\underline{a} = \underline{55,0} \text{ mm.}$$

$$\underline{\lg} = \frac{55}{1,5450431} = \underline{35,6} \text{ mm.}$$

Radio "b" de la esfera tangente a las aristas

Aplicando la fórmula general [3] (ver lám. 33)

$$\boxed{b} = \sqrt{a^2 - \frac{\rho_n^2}{4}} = \sqrt{\frac{1}{4} \times \frac{1 + \operatorname{sen}^2(\pi:n)}{\operatorname{sen}^2(\pi:n)} \times \rho^2 - \frac{\rho_n^2}{4}} =$$

$$= \sqrt{\frac{1}{4} \left(\frac{1 + \operatorname{sen}^2(\pi:n)}{\operatorname{sen}^2(\pi:n)} - 1 \right) \times \rho_n} = \sqrt{\frac{1}{4 \operatorname{sen}^2(\pi:n)}} \times \rho_n = \boxed{\frac{1}{2 \operatorname{sen}(\pi:n)} \times \rho_n}$$

Para el caso del dibujo, $n = 9$, será: $\frac{\pi}{9} = \frac{180^\circ}{9} = 20^\circ$

$$\lg 1 = 0,0000000$$

$$\lg 2 = 0,3010300$$

$$\lg \operatorname{sen} 20^\circ = -\bar{7},5340517 + = \bar{7},8350817 -$$

$$\underline{b} = \text{antilog } 0,1649183 \times \rho_n = \underline{1,4619020} \times \rho_n$$

$$= \underline{52,0} \text{ mm} = \underline{b}$$

Handwritten text in Chinese, likely a ledger or account book. The text is arranged in vertical columns, typical of traditional Chinese writing. The characters are somewhat faded and difficult to read precisely, but the layout suggests a structured record of transactions or accounts.

Radio " d_4 " de la circunferencia circunscrita a una cara cuadrada de lado " l_n "

Se demuestra en Geometría, es:

$$d_4 = \frac{\sqrt{2}}{2} l_n = 0,70710678... l_n$$

Para el caso del dibujo, $n = 9$, es $l_9 = 35,6 \text{ mm}$, y

$$d_4 = 0,70710678... \times 35,6 = 25,2 \text{ mm}$$

Radio " d_n " de la circunferencia circunscrita a una cara regular de " n " lados.

En la lám. 46, h2, hemos determinado su valor, que es

$$d_n = \frac{1}{2 \operatorname{sen}(\pi/n)} \cdot l_n = b$$

La propiedad $d_n = b$ se deduce también de la figura de la lámina 47.

Para el caso del dibujo, $n = 9$, $l_9 = 35,6 \text{ mm}$, será

$$d_9 = b = 52,0 \text{ mm}.$$

Radio " C_4 " de la esfera tangente a las caras cuadradas de lado " l_n "

Aplicando la fórmula general [2] (ver lám. 33)

Mathematics

1. The first part of the document discusses the importance of mathematics in various fields.

2. The second part of the document discusses the importance of mathematics in various fields.

$$E = mc^2$$

3. The third part of the document discusses the importance of mathematics in various fields.

4. The fourth part of the document discusses the importance of mathematics in various fields.

5. The fifth part of the document discusses the importance of mathematics in various fields.

6. The sixth part of the document discusses the importance of mathematics in various fields.

$$\begin{aligned}
 \boxed{C_4} &= \sqrt{a^2 - (d_4)^2} = \sqrt{\frac{1}{4} \times \frac{1 + \operatorname{sen}^2(\pi:n)}{\operatorname{sen}^2(\pi:n)} \times (\ell_n)^2 - \frac{1}{2} (\ell_n)^2} = \\
 &= \sqrt{\frac{1}{4} \left(\frac{1 + \operatorname{sen}^2(\pi:n)}{\operatorname{sen}^2(\pi:n)} - 2 \right) \times \ell_n} = \sqrt{\frac{1}{4} \left(\frac{1 - \operatorname{sen}^2(\pi:n)}{\operatorname{sen}^2(\pi:n)} \right) \times \ell_n} = \\
 &= \sqrt{\frac{1}{4} \times \frac{\cos^2(\pi:n)}{\operatorname{sen}^2(\pi:n)}} \times \ell_n = \sqrt{\frac{1}{4 \operatorname{tg}^2(\pi:n)}} \times \ell_n = \boxed{\frac{1}{2 \operatorname{tg}(\pi:n)} \ell_n}
 \end{aligned}$$

Para el caso del dibujo, $n=9$; $\operatorname{tg}(120^\circ:9) = \operatorname{tg} 20^\circ$; $\ell_9 = 35,6$

$$C_4 = \frac{1}{2 \operatorname{tg} 20^\circ} \times \ell_9 =$$

$$\begin{aligned}
 \operatorname{tg} 1^\circ &= 0,00000000 \\
 \operatorname{tg} 2^\circ &= 0,30103000 \\
 \operatorname{tg} 20^\circ &= 7,5610659 \dots + = 7,8620956 - \\
 C_4 &= \text{antilog } 0,1379044 \times \ell_n = \\
 &= 1,3737396 \dots \times 35,6 = 48,9 \text{ mm.} = C_4
 \end{aligned}$$

Radio "C_n" de la esfera tangente a las caras regulares de "n" lados

Aplicando la fórmula general [2] (ver lám. 33)

$$\begin{aligned}
 \boxed{C_n} &= \sqrt{a^2 - (d_n)^2} = \sqrt{\frac{1}{4} \times \frac{1 + \operatorname{sen}^2(\pi:n)}{\operatorname{sen}^2(\pi:n)} \times (\ell_n)^2 - \frac{1}{4 \operatorname{sen}^2(\pi:n)} \times (\ell_n)^2} \\
 &= \sqrt{\frac{1}{4} \left(\frac{1 + \operatorname{sen}^2(\pi:n)}{\operatorname{sen}^2(\pi:n)} - \frac{1}{\operatorname{sen}^2(\pi:n)} \right) \times \ell_n} = \sqrt{\frac{1}{4} \left(\frac{\operatorname{sen}^2(\pi:n)}{\operatorname{sen}^2(\pi:n)} \right) \times \ell_n} =
 \end{aligned}$$

$$= \boxed{\frac{1}{2} l_n}$$

valor que se deduce directamente de la figura de la lámina.

Para el caso del dibujo, $n = 9$, $l_9 = 35,6 \text{ mm}$, será:

$$c_n = \frac{1}{2} \times 35,6 = \underline{\underline{17,8 \text{ mm}}}$$

Ángulo rectilíneo " α_n " del diedro formado por una cara cuadrada, con el plano diametral del arquimedianos que pasa por una arista de aquella.

Se obtiene, en función de su tangente, por la fórmula general [5] (ver lám. 33).

$$\boxed{\operatorname{tg} \alpha_n} = \frac{2 c_n}{\sqrt{4 (d_n)^2 - (l_n)^2}} = 2 \times \frac{1}{2 \operatorname{tg} (\pi : n)} \times l_n : \sqrt{4 \times \frac{1}{4} (l_n)^2 - (l_n)^2} =$$

$$= \frac{l_n}{\operatorname{tg} (\pi : n)} : l_n = \frac{1}{\operatorname{tg} (\pi : n)} = \boxed{\operatorname{ctg} (\pi : n)}$$

Este resultado puede obtenerse directamente de la figura de la lámina.

Ángulo rectilíneo " α_n " del diedro formado por una cara regular de " n " lados, con el plano diametral del arquimedianos que pasa por una arista de aquella.

Se obtiene, en función de su tangente, por la fórmula general [5] (ver lám. 33).

The first part of the paper is devoted to the study of the properties of the function $f(x)$ defined by the equation $f(x) = \frac{1}{x}$. It is shown that $f(x)$ is a decreasing function on the interval $(0, \infty)$ and that it has a horizontal asymptote at $y = 0$ and a vertical asymptote at $x = 0$.

In the second part, we consider the function $g(x) = \ln x$. It is shown that $g(x)$ is an increasing function on the interval $(0, \infty)$ and that it has a vertical asymptote at $x = 0$. The function $g(x)$ is also shown to be concave down on the interval $(0, \infty)$.

The third part of the paper is devoted to the study of the function $h(x) = \frac{1}{x^2}$. It is shown that $h(x)$ is a decreasing function on the interval $(0, \infty)$ and that it has a horizontal asymptote at $y = 0$ and a vertical asymptote at $x = 0$. The function $h(x)$ is also shown to be concave up on the interval $(0, \infty)$.

Finally, we consider the function $k(x) = \frac{1}{x^3}$. It is shown that $k(x)$ is a decreasing function on the interval $(0, \infty)$ and that it has a horizontal asymptote at $y = 0$ and a vertical asymptote at $x = 0$. The function $k(x)$ is also shown to be concave up on the interval $(0, \infty)$.

$$\boxed{\tan \alpha_n} = \frac{2 c_n}{\sqrt{4 (d_n)^2 - (l_n)^2}} = \frac{2 \times \frac{1}{2} l_n}{\sqrt{4 \times \frac{1}{4 \sin^2 (\pi : n)} \times (l_n)^2 - (l_n)^2}} =$$

$$= \frac{1}{\sqrt{\frac{1}{\sin^2 (\pi : n)} - 1}} = \frac{1}{\sqrt{\frac{1 - \sin^2 (\pi : n)}{\sin^2 (\pi : n)}}} = \sqrt{\frac{\sin^2 (\pi : n)}{\cos^2 (\pi : n)}} = \boxed{\tan (\pi : n)}$$

Este resultado puede obtenerse directamente de la figura de la lámina.

Ángulo rectilíneo " φ_{4-n} " del diedro formado por una cara cuadrada y otra regular de " n " lados

Aplicando la fórmula general [4] (lámn. 33)

$$\varphi_{4-n} = \alpha_4 + \alpha_n = \quad \text{de donde}$$

$$\boxed{\tan (\varphi_{4-n})} = \tan (\alpha_4 + \alpha_n) = \frac{\tan \alpha_4 + \tan \alpha_n}{1 - \tan \alpha_4 \tan \alpha_n} = \frac{\tan (\pi : n) + \tan (\pi : n)}{1 - \tan (\pi : n) \tan (\pi : n)}$$

$$= \frac{\tan (\pi : n) + \tan (\pi : n)}{1 - 1} = \boxed{\infty} \quad \text{para cualquier valor ociente de "n"}$$

7 por consiguiente, siempre será:

$$\boxed{\varphi_{4-n} = 90^\circ}$$

Esta propiedad se deduce directamente de la figura de la lámina (estos arquimedianos son prismas rectos regulares de bases P_n y caras laterales cuadradas).

THE UNIVERSITY OF CHICAGO

THE UNIVERSITY OF CHICAGO

THE UNIVERSITY OF CHICAGO

THE UNIVERSITY OF CHICAGO

THE UNIVERSITY OF CHICAGO

THE UNIVERSITY OF CHICAGO

THE UNIVERSITY OF CHICAGO

THE UNIVERSITY OF CHICAGO

THE UNIVERSITY OF CHICAGO

THE UNIVERSITY OF CHICAGO

THE UNIVERSITY OF CHICAGO

THE UNIVERSITY OF CHICAGO

Ángulo rectilíneo " φ_{4-n} " del diedro formado por dos caras cuadradas

Aplicando la fórmula general [4] (lám. 33)

$$\varphi_{4-n} = 2 \alpha_4 \quad \text{de donde}$$

$$\boxed{\frac{1}{t_2} \varphi_{4-n}} = \frac{1}{t_2} (2 \alpha_4) = \frac{2 \frac{1}{t_2} \alpha_4}{1 - \frac{1}{t_2}^2 \alpha_4} = \frac{2 \operatorname{ctg}(\pi:n)}{1 - \operatorname{ctg}^2(\pi:n)} = \frac{2}{\frac{1}{\operatorname{ctg}(\pi:n)} - \operatorname{ctg}(\pi:n)}$$

$$= \frac{2}{\operatorname{tg}(\pi:n) - \frac{1}{\operatorname{tg}(\pi:n)}} = \frac{2}{\frac{\operatorname{tg}^2(\pi:n) - 1}{\operatorname{tg}(\pi:n)}} = \frac{2 \operatorname{tg}(\pi:n)}{-(1 - \operatorname{tg}^2(\pi:n))} =$$

$$= - \frac{2 \operatorname{tg}(\pi:n)}{1 - \operatorname{tg}^2(\pi:n)} = - \frac{\operatorname{tg}(2 \times (\pi:n))}{1} = \boxed{- \operatorname{tg}(2\pi:n)}$$

∴ de esta última

$$\boxed{\varphi_{4-n}} = \pi - \frac{2\pi}{n} = \frac{\pi n - 2\pi}{n} = \boxed{\pi - \frac{n-2}{n}}$$

Este resultado puede obtenerse directamente de la figura de la lámina

Para el caso del dibujo, $n = 9$, será

$$\underline{\varphi_{4-n}} = 180^\circ \times \frac{9-2}{9} = \underline{140^\circ}$$



Área lateral "S" del arquimediano

Se compone de la suma de " n " caras cuadradas de

Handwritten text in the upper middle section of the page.

Handwritten text in the middle section of the page.

Handwritten text in the middle section of the page.

Handwritten text in the middle section of the page.

Handwritten text in the lower middle section of the page.

Handwritten text in the lower middle section of the page.



Handwritten text in the bottom section of the page.

tado " l_n " y 2 caras regulares de " n " lados y de igual magnitud " l_n ".

La apotema de una cara regular de " n " lados, en función de su lado " l_n ", (ver fórmula [2], lám 46, hoja 4), es

$$k_n = \frac{1}{2 \tan \left(\frac{\pi}{n} \right)} l_n$$

el área total será pues:

$$\begin{aligned} [S] &= n (l_n)^2 + \frac{n l_n}{2} \times \frac{1}{2 \tan \left(\frac{\pi}{n} \right)} \times l_n = \left(n + \frac{n}{4 \tan \left(\frac{\pi}{n} \right)} \right) (l_n)^2 = \\ &= \left(1 + \frac{1}{4 \tan \left(\frac{\pi}{n} \right)} \right) n (l_n)^2 = \boxed{\left(1 + \frac{1}{4} \cotg \left(\frac{\pi}{n} \right) \right) n (l_n)^2} \end{aligned}$$

Volumen "V" del arquimedianos

Se compone de la suma de " n " pirámides regulares de base cuadrada y altura " c_n ", y de 2 pirámides de base regular de " n " lados y altura " c_n ".

su valor será pues:

$$\begin{aligned} [V] &= n (l_n)^2 \times \frac{1}{2 \tan \left(\frac{\pi}{n} \right)} \times \frac{l_n}{3} + 2 \times \frac{n l_n}{2} \times \frac{1}{2 \tan \left(\frac{\pi}{n} \right)} \times l_n \times \frac{l_n}{2 \times 3} = \\ &= \left(\frac{1}{6 \tan \left(\frac{\pi}{n} \right)} + \frac{1}{12 \tan \left(\frac{\pi}{n} \right)} \right) n (l_n)^3 = \frac{1}{4 \tan \left(\frac{\pi}{n} \right)} n (l_n)^3 = \\ &= \boxed{\frac{1}{4} \cotg \left(\frac{\pi}{n} \right) n (l_n)^3} \end{aligned}$$

$$\frac{1}{2} \frac{d}{dt} \left(\frac{1}{r^2} \right) = -\frac{1}{r^3} \frac{dr}{dt}$$

$$\frac{d}{dt} \left(\frac{1}{r^2} \right) = -\frac{2}{r^3} \frac{dr}{dt}$$

$$\frac{d}{dt} \left(\frac{1}{r^2} \right) = -\frac{2}{r^3} \frac{dr}{dt}$$

$$\frac{d}{dt} \left(\frac{1}{r^2} \right) = -\frac{2}{r^3} \frac{dr}{dt}$$

$$\frac{d}{dt} \left(\frac{1}{r^2} \right) = -\frac{2}{r^3} \frac{dr}{dt}$$

$$\frac{d}{dt} \left(\frac{1}{r^2} \right) = -\frac{2}{r^3} \frac{dr}{dt}$$

$$\frac{d}{dt} \left(\frac{1}{r^2} \right) = -\frac{2}{r^3} \frac{dr}{dt}$$

FIGURA CORPÓREA

Se obtiene por acoplamiento de "n" cuadrados de lado " l_n " y de 2 caras regulares de "n" lados, de igual magnitud " l_n ". El acoplamiento deberá hacerse de forma que en cada vértice concurren 2 cuadrados y un polígono regular de "n" lados.

En el cuadro sinóptico que damos a continuación, se resumen los resultados analíticos obtenidos anteriormente.

CUADRO SINÓPTICO

Magnitud	Valor exacto	Valor decimal aproximado
a	$\frac{1}{2} \sqrt{\frac{1 + \operatorname{sen}^2 (\pi : n)}{\operatorname{sen}^2 (\pi : n)}} l_n$	Variable con "n"
b	$\frac{1}{2 \operatorname{sen} (\pi : n)} l_n$	Variable con "n"
c_4	$\frac{1}{2 \operatorname{tg} (\pi : n)} l_n$	Variable con "n"
c_n	$\frac{1}{2} l_n$	0,50 00 00... l_n
d_4	$\frac{\sqrt{2}}{2} l_n$	0,70 71 07... l_n
d_n	$\frac{1}{2 \operatorname{sen} (\pi : n)} l_n$	Variable con "n"
m	$\sqrt{\frac{1}{1 + \operatorname{sen}^2 (\pi : n)}} l_n$	Variable con "n"
α_4	$\operatorname{tg} \alpha_4 = \operatorname{ctg} (\pi : n)$	$\alpha_4 = \operatorname{arc} \operatorname{ctg} (\pi : n)$ Variable con "n"
α_n	$\operatorname{tg} \alpha_n = \operatorname{tg} (\pi : n)$	$\alpha_n = \operatorname{arc} \operatorname{tg} (\pi : n)$ Variable con "n"
φ_{4-n}	$\frac{\pi}{2}$	$\varphi_{4-n} = 90^\circ$
φ_{4-n}	$\pi \times \frac{n-2}{n}$	Variable con "n"
S	$\left[1 + \frac{1}{4} \operatorname{ctg} (\pi : n) \right] n (l_n)^2$	Variable con "n"
V	$\frac{1}{4} \operatorname{ctg} (\pi : n) n (l_n)^3$	Variable con "n"

PROCESO GRÁFICO - ANALÍTICO

Después del cálculo de las magnitudes principales, vamos a proceder en la lámina 47, a la representación gráfica de un Arquimedianos de la Serie B_n, en el caso particular de ser $n=9$.

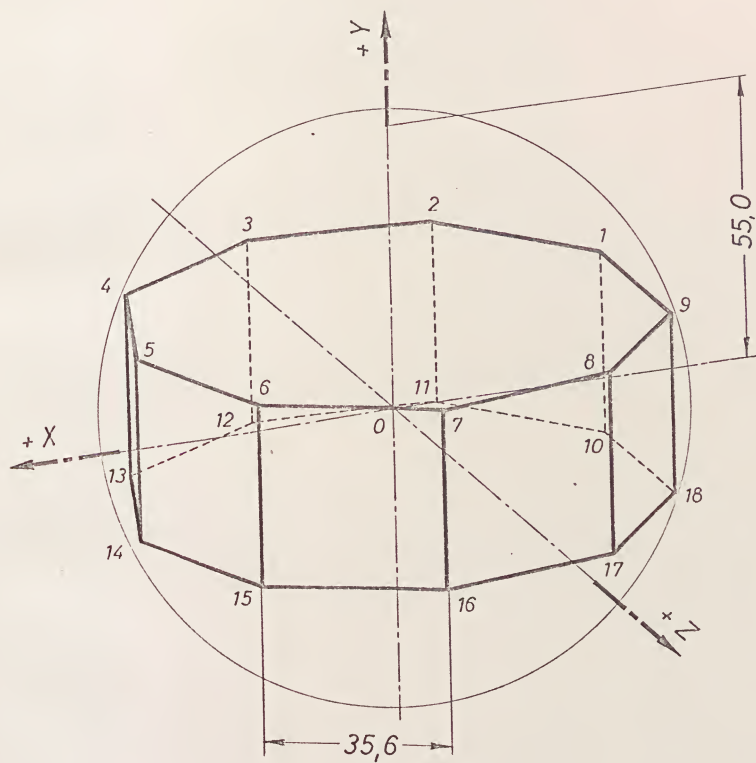
Para su trazado nos valdremos de las cotas calculadas por las fórmulas anteriores, determinadas previamente para $n=9$. Dichas magnitudes las obtendremos en función todo " l_9 " del arquimedianos, cuya longitud es de 35,6 mm.

El cálculo de dichas magnitudes se efectúa a continuación:

$$\begin{aligned}
 l_9 &= \text{Dato del ejercicio} &= 35,6 \text{ mm} \\
 a &= 1,54 \ 50 \ 43 \ 1 \times 35,6 &= 55,0 \text{ mm} \\
 b &= 1,46 \ 19 \ 03 \ 0 \times 35,6 &= 52,0 \text{ mm} \\
 c_h &= 1,37 \ 37 \ 29 \ 6 \times 35,6 &= 48,9 \text{ mm} \\
 c_g &= 0,5 &\times 35,6 &= 17,8 \text{ mm} \\
 d_h &= 0,70 \ 71 \ 06 \ 8 \times 35,6 &= 25,2 \text{ mm} \\
 d_g &= 1,46 \ 19 \ 03 \ 0 \times 35,6 &= 52,0 \text{ mm}
 \end{aligned}$$

Con ayuda de estas cotas es fácil obtener las proporciones del arquimedianos sobre I, II y III, ya que éste tiene la forma de un prisma recto regular, cuyas bases son de " n " lados, y altura " l_n ".

En la lámina 47 hemos situado el arquimedianos, con sus bases paralelas a II y su centro en O. El polígono de la base se puede dibujar fácilmente, conocidos el radio de la circunferencia circunscrita $b = 52,0 \text{ mm}$ y la longitud de su lado $l_9 = 35,6 \text{ mm}$.

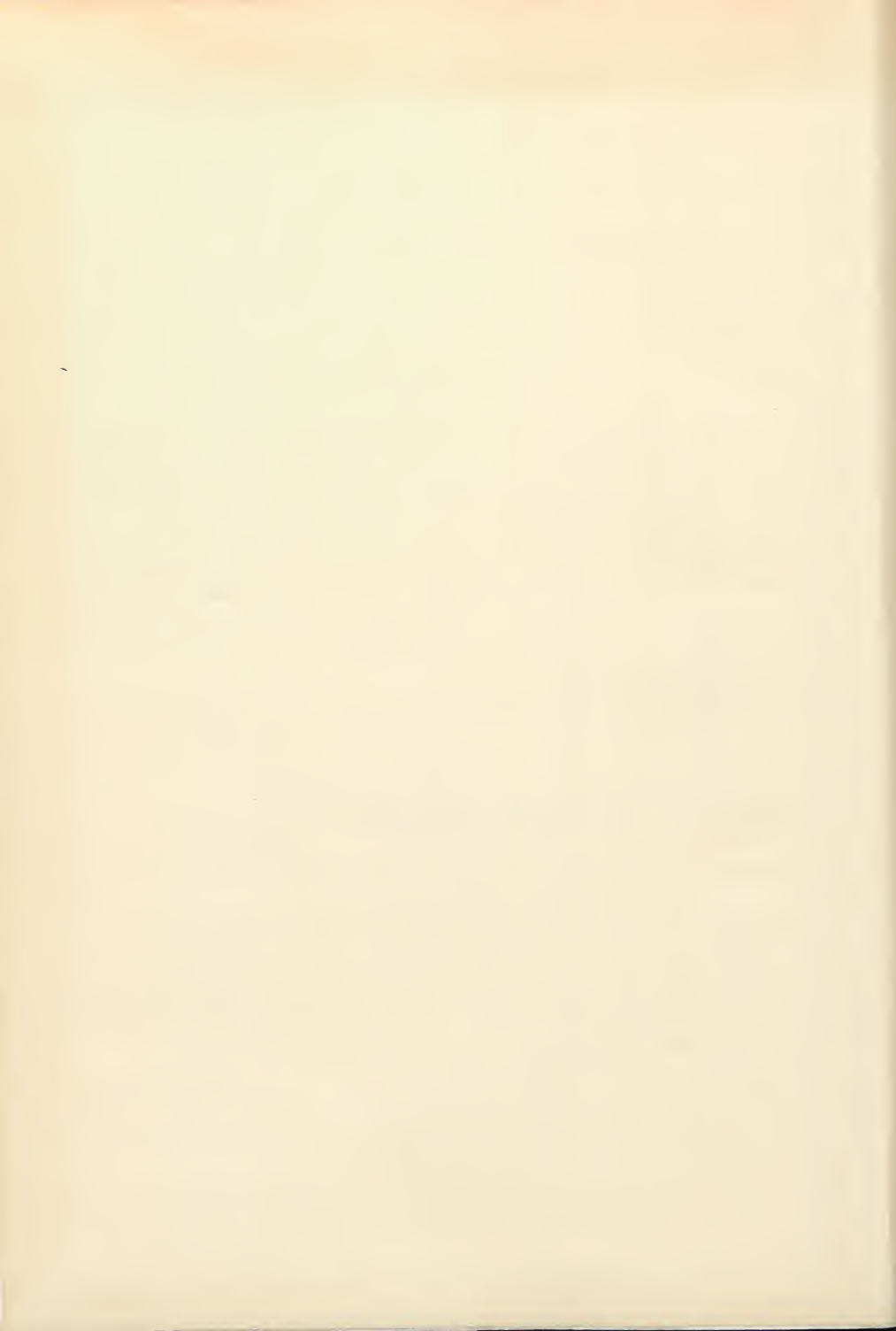


$$n = 9$$

Arquimedianos Serie B_n









UNIVERSIDAD DE SEVILLA



600107505

ESCUELA UNIVERSITARIA DE INGENIERIA TECNICA
INDUSTRIAL DE SEVILLA

ESTANTE.....

T A B L A.....

NUMERO 7815

